A DYNAMIC MODEL FOR THE ALLOCATION OF AIRSTRIKES AGAINST A LINES-OF-COMMUNICATION NETWORK

by

Larry Reginald Capps

United States
Naval Postgraduate School

THESIS

A DYNAMIC MODEL FOR THE ALLOCATION OF
AIRSTRIKES AGAINST A LINES-OF-COMMUNICATION NETWORK

by

Larry Reginald Capps

SEP 1970

A Dynamic Model for the Allocation of Airstrikes Against a Lines-of-Communication Network

by

Larry Reginald Capps
Major, United States Army
B.S., United States Military Academy, 1963

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the
NAVAL POSTGRADUATE SCHOOL
September 1970

ABSTRACT

A computer model is presented for determining the daily allocation of airstrikes for interdicting a lines-of-communication (LOC) network assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is restricted by network capacity and (2) parameter values for each arc are independent. Upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc are required as input data. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure is programmed in daily cycles, allowing repair on interdicted arcs. A sample problem and all documentation necessary for duplicating the computer program are given.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

Figure

TABLE OF SYMBOLS AND ABBREVIATIONS

(i,j)	arc between nodes i and j
C_{ij}	cost of one unit of flow on arc (i.j)
CRT_{ij}	cumulative repair time for arc (i,j)
d_{ij}	damage on arc (i.j) caused by n_{ij} strikes
D_{ij}	total damage on arc (i,j) over one or more days
F'	cost of a minimum cost circulation flow after arc (i,j) has been struck
k_{ij}	cost to repair the arc damage caused by n_{ij} strikes
K_{ij}	maximum cost to restore an arc from its lower flow bound to its upper flow bound
L_{ij}	lower flow bound on arc (i,j)
M_{ij}	actual flow on arc (i,j)
n_{ij}	the number of strikes allocated to arc (i,j) in one day
N	maximum number of strikes that can be allocated in any one day
r_{ij}	amount of repair completed on arc (i,j) in one day
s	the Lagrange Multiplier
t_{ij}	time to repair the arc damage caused by n_{ij} strikes
T_{ij}	maximum time to restore an arc from its lower flow bound to its upper flow bound
U_{ij}	initial upper flow bound (maximum capacity) on arc (i,j)
$\bar{U}_{ij}$	upper flow bound on arc (i,j) at any time
V_{ij}	vulnerable flow capacity of arc (i,j), i.e., $V_{ij} = U_{ij} - L_{ij}$
VP_{ij}	vulnerability parameter for arc (i,j)

I. INTRODUCTION

A. GENERAL

The ability of a military force to conduct sustained combat operations any distance from its base of supply may be limited by the land lines-of-communication (LOC) network over which its supplies must flow. This is particularly true if air and sea resupply are not feasible, or the opposing force has superiority over these alternatives. The amount and type of material required to support a military force are determined by the nature of combat operations and the intensity of combat. A large conventional force would require more pounds-per-man-day than a small guerilla force capable of obtaining a portion of its requirements in the area in which it operates. Undoubtedly, however, a requirement will exist for supplies in excess of what can be obtained locally or what can be transported organically by the combat force.

The land LOC network then becomes an important military target. Flow of military material over the network can be reduced by any of three alternatives: (1) attacking the transport capability, i.e., vehicles or rolling stock, (2) attacking manufacturing and supply centers, thereby reducing the amount of materiel available, or (3) attacking the road/rail LOC network over which the materiel must flow. Certainly, a combination of these alternatives would be most desirable. However, political considerations might preclude attacking manufacturing and supply centers; and terrain, weather, and lack of intelligence information might limit the effectiveness of attempting

to attack convoys moving over the network.

Such a scenario is particularly applicable to the present war in Southeast Asia and is a general characteristic of limited warfare. The North Vietnamese neither possess a sophisticated air resupply capability, nor can they openly move supplies by sea to support their forces operating in South Vietnam. They do, however, enjoy immunity from attack on all their facilities north of the 17th Parallel. Therefore, the supply of materiel to their forces operating in the South is primarily limited by the transportation system over which this materiel must flow.

The objective of an interdiction campaign by airstrike must then be to systematically reduce the capacity of road or rail segments which will ultimately restrict the amount of materiel that can flow through the network.

B. AVAILABLE LITERATURE

Several authors have published works on the problem of interdicting a lines-of-communication network. Durbin [3] has constructed a model for determining maximum cargo flow as a function of available transport vehicles. His method sequentially selects and interdicts the most vital segment in the network until flow is reduced to zero or a specific number of segments have been destroyed. The model operates in daily cycles and allows repair on interdicted segments.

Wollmer [11, 14, 15] has developed a procedure to determine the n most vital segments in a network and to evaluate the effect

of removing the most vital segment. In the latter work, road capacity as a function of the number of strikes is reduced (1) by a known quantity and then (2) by a quantity that is a random variable.

Wollmer [12, 13] has also developed algorithms and a model for evaluating a targeting strategy whose aim is the greatest reduction in network flow. The model is programmed in daily cycles with the user specifying the number of days and strikes.

Mustin [9] uses the topological dual to represent a lines-of-communication network and assumes the decrease in capacity of a road or rail segment has a fixed deterministic rate which is linear between its upper and lower bounds.

Nugent [10] also uses the topological dual for network representation, but assumes the reduction in capacity of a road or rail segment is exponential with decreasing marginal returns. His model is constrained by the number of available sorties and does not allow for the dynamics of the daily allocation problem.

C. ALLOCATION OF EFFORT

Many techniques have been devised to aid in solving the problem of allocating limited resources. Hancock [8] provides a reference on classical optimization techniques; Danskin [1, 2] considers the theory of maxima and minima and its application to weapons systems allocation; and Koopman [6, 7] considers a similar problem in the allocation of search and screening effort. The problem facing the strike planner is to determine what road

or rail segments in the network to attack and how many strikes to allocate to each selected segment. Numerous considerations must be weighed in order to make this determination: (1) the number of sorties available on a given day, (2) the probability that a strike against a particular segment is successful, (3) the expected cost to the enemy in time and effort to repair an interdicted segment, (4) the expected cost of aircraft lost in striking a defended segment, and (5) the expected reduction in capacity if a strike is successful.

The obvious procedure in making the allocation of N sorties is to attack those segments (at a level of n_{ij} each) that maximize the ratio of benefits to cost. As will be shown later, Gibbs' Lemma [2] provides the mathematical basis for determining this allocation.

II. OBJECTIVE AND SCOPE

The objective of this paper is to present a model for determining the daily allocation of airstrikes for interdicting a lines-of-communication network assuming an exponential damage function. The model seeks to allocate a maximum of N (specified) sorties per day to those arcs (segments) which yield the greatest reduction in flow capacity and the greatest time and cost to repair. Two options are provided that will allow the strike planner to (1) specify the number of strikes per day and the number of days in the interdiction campaign, or (2) select a percentage of the maximum possible flow to be reduced. Using the second option, the maximum available number of sorties per day and the number of campaign days are specified.

Certain information is assumed to be available to the strike planner for each arc in the LOC network:

(1) Upper flow bound - maximum capacity in units per day.

(2) Lower flow bound - minimum capacity in units per day below which the flow cannot be reduced.

(3) Maximum repair time - the time required to repair the maximum possible damage on an arc.

(4) Maximum repair cost - the cost to repair the maximum possible damage on an arc.

(5) Vulnerability parameter - the probability that a sortie is successful. The strike planner can manipulate this parameter by providing his own definition of "successful."

The vulnerability of any arc, V_{ij}, is defined as the upper flow bound minus the lower flow bound, i.e., $U_{ij}-L_{ij}$. A necessary assumption is that the vulnerability, repair cost, and repair time are independent of the same parameters for any other arc.

By assuming an exponential damage function, the reduction in capacity d_{ij}, for any arc (i,j) caused by n_{ij} strikes is:

$$d_{ij} = V_{ij}[1 - \exp(-VP_{ij}\ n_{ij})] \tag{1}$$

Arc repair time and cost are linear functions of the amount of damage caused by n_{ij} strikes:

$$t_{ij} = (d_{ij}/V_{ij})T_{ij} \tag{2}$$

$$k_{ij} = (d_{ij}/V_{ij})K_{ij} \tag{3}$$

The amount of arc repair that can be completed in any one day is a function of the computed cumulative arc repair time and the total damage on the arc:

$$r_{ij} = (1/CRT_{ij})\ D_{ij} \tag{4}$$

The value of allocating n_{ij} strikes per day to arc (i,j) is determined by:

$$\text{Value}_{ij} = (F' - F)\ t_{ij} + k_{ij} \tag{5}$$

The model daily selects and interdicts those arcs that maximize (5), performs repair as indicated by (4), and steps to the next day. The procedure has been programmed in FORTRAN IV for use on the IBM 360/67 computer.

III. NETWORK REPRESENTATION OF LINES-OF-COMMUNICATION

A. NETWORK STRUCTURE

A transportation system, whether single or multimode, can be represented by a connected graph of nodes and directed arcs. For example, a highway system would consist of roads (arcs) and the intersection of roads (nodes). If a road is one-way, the beginning node of an arc is the point where traffic enters the segment and the ending node is the point where traffic leaves the segment. If a road is two-way, it is represented by two arcs, one in each direction.

The LOC user attempts to send units of flow, expressible in vehicles per day, tons per day, etc., from the source to the sink. The source of flow could be a rear-area supply center and the sink could be the battlefront or a forward-area supply center. If the user is constrained by the capacity of the transportation system, it is assumed that he will attempt to maximize the total flow of cargo through the network at a minimum cost.

Consider the example LOC networ shown in Fig. 1. Suppose the user wishes to send flow from node AA to node ZZ. Node AA is the source and node ZZ is the sink. The route (AA,A,K,L,ZZ) is one of many possible routes over which the flow can be sent. Associated with each arc are certain parameters: lower bound on flow, upper bound on flow (flow capacity), and the cost per unit flow. For arc (K,L) the following parameters are given:

Lower bound (L_{ij})	......	0
Upper bound (U_{ij})	......	400
Flow cost (C_{ij})	......	7

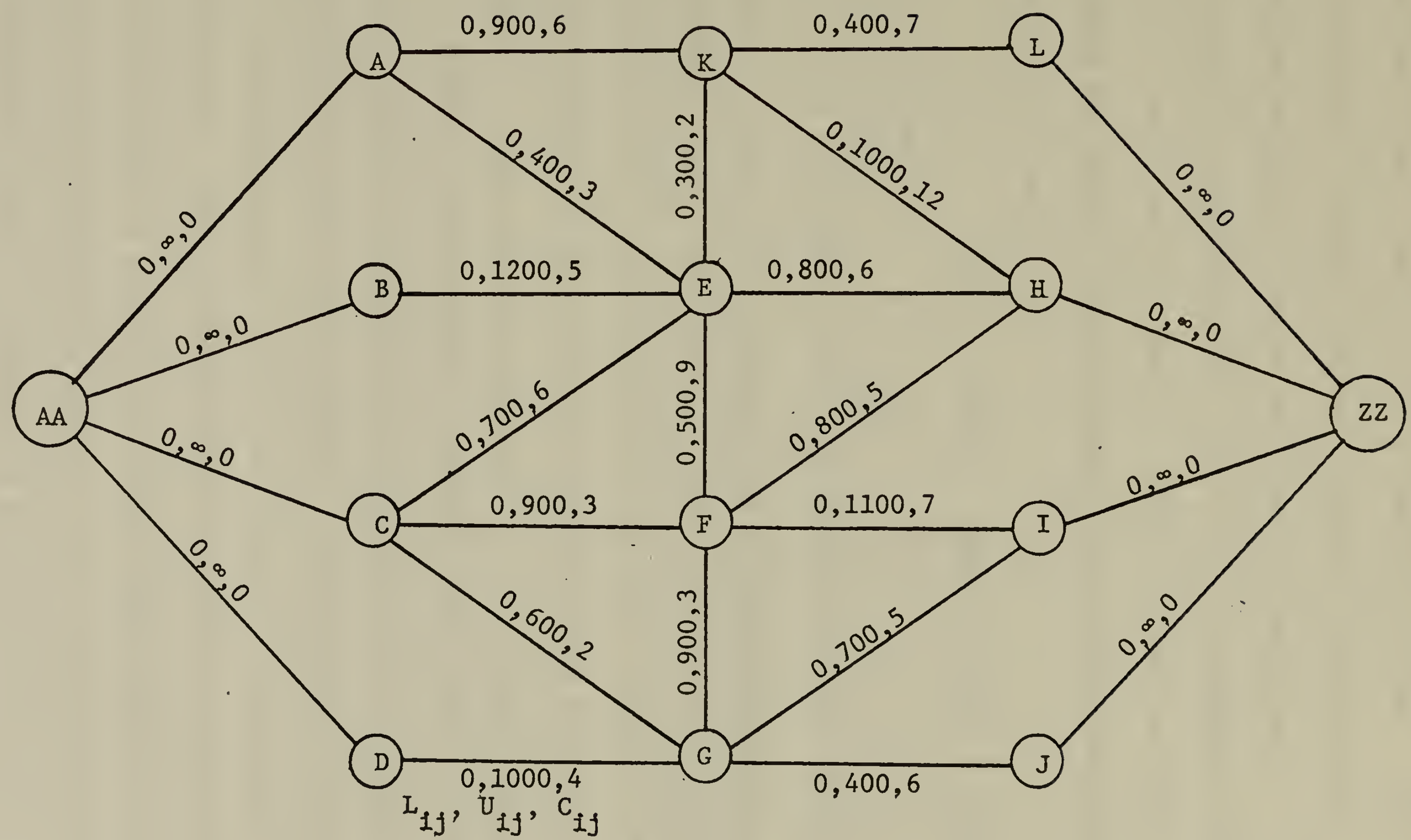


Figure 1. An Example Network

These parameters imply that between 0 and 400 units of flow per time unit can be sent over (K,L) at a cost of 7 per unit. This cost can be expressed in terms of dollars per ton of cargo, dollars per vehicle, manpower units, or any other definable cost. If the arc is used to its capacity, i.e., $M_{ij} = U_{ij}$, it would cost 2800 per time unit to send flow over this arc. The total cost per time unit of sending 400 units of flow from node AA to node ZZ over route (AA,A,K,L,ZZ) would be:

$$\sum_{\substack{\text{all } (i,j) \\ \text{in} \\ \text{the route}}} M_{ij}C_{ij} = 400(0) + 400(6) + 400(7) + 400(0) = 4200$$

It is important to note that the flow out of a node equals the flow into a node. This conservation of flow implies there can be no storage at nodes. Therefore, for most general networks, if x units of flow are sent from the source to the sink, then any flow pattern that accomplishes this will have net flows of x units out of the source, x units into the sink, and zero on all other nodes.

In actual LOC networks, there may be more than one source and/or sink. This can be handled, provided any source can supply any sink, by adding artificial nodes and arcs. This allows the user to convert the multi-source, multi-sink problem to a single-source, single-sink problem. The artificial arcs are used to connect the true sources (sinks) to an artificial super source (sink). Note in Figure 1 that arcs (AA,A), (AA,B), (AA,C), and (AA,D) can be considered artificial arcs connecting the true sources A, B, C, and D to the single artificial super source AA. Similarly, arcs (L,ZZ),

(H,ZZ), (I,ZZ), and (J,ZZ) can be considered artificial arcs connecting the true sinks L, H, I, and J to the artificial super sink ZZ. The flow on an artificial arc, e.g., (AA,A), can be considered the rate that cargo leaves the true source A. The capacity for artificial arcs is usually infinite, but could represent the maximum supply rate at a facility such as a supply center or warehouse. The lower flow bound and cost on artificial arcs are normally zero. Artificial arcs are not considered vulnerable to attack.

Actual flow on an arc is designated M_{ij} if the flow is from node i to node j and M_{ji} if the flow is in the reverse direction. Normally, flow is assumed to be from the source to the sink but can be in either direction on intermediate two-way arcs.

U_{ij}, $\bar{U}_{ij}$, and L_{ij} represent the upper flow bound, the flow capacity at any time, and the lower flow bound, respectively, for any arc (i,j). The following relationship must hold for these parameters:

$$0 \le L_{ij} \le M_{ij} \le \bar{U}_{ij} \le U_{ij}$$

The maximum possible flow through the network can be determined by using the maximal-flow minimal-cut theorem [4]. It states that the maximum flow in a network is equal to the minimum value of all cut-sets, where a cut-set is defined as a set of arcs that separate the source from the sink. The value of a cut-set is defined as the sum of the capacities of all arcs in the cut-set. In the example network, Figure 2, arcs (A,K), (K,E), (E,H), (E,F), (C,F), (C,G), and (D,G) form the minimum cut-set with a value of 5000 units.

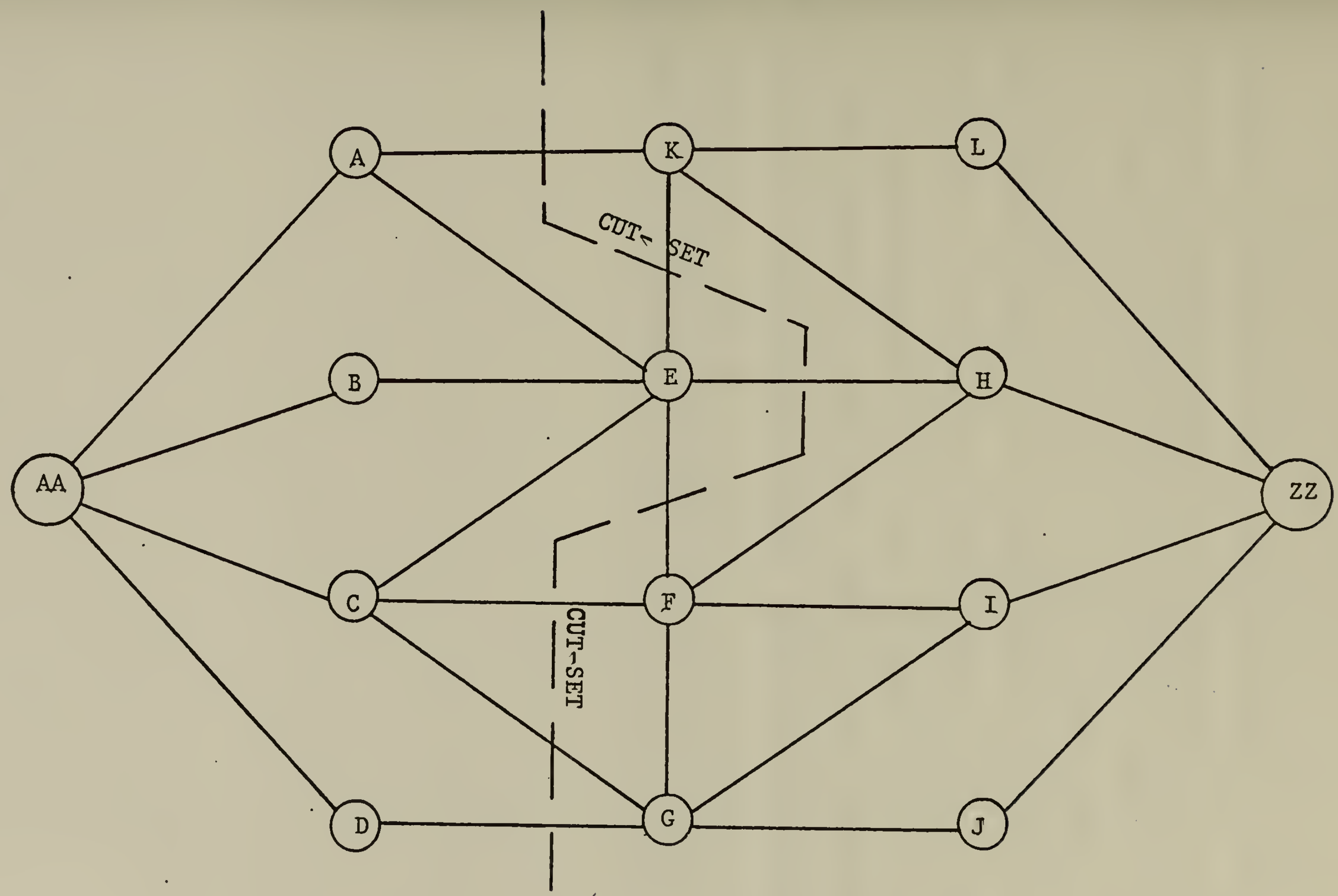


Figure 2. Minimum Cut-Set for Example Network

B. CIRCULATION FLOWS IN NETWORKS

As previously stated, if the LOC user is constrained by the capacity of the network, it is assumed he wishes to maximize the total flow through the network at a minimum cost. Such a problem is efficiently handled by the "Out-of-Kilter" algorithm developed by Fulkerson [4]. The network is restructured by adding a "universal" arc from the super sink to the super source as in Fig. 3. This artificial arc is assigned an upper flow bound greater than or equal to the maximum possible flow through the network (usually infinite), a lower bound of zero, and a unit flow cost less than the flow cost on any arc (usually minus infinity). Hence, the problem of maximizing source-sink flow is converted to one of finding a minimum cost circulation flow by maximizing flow on the universal arc.

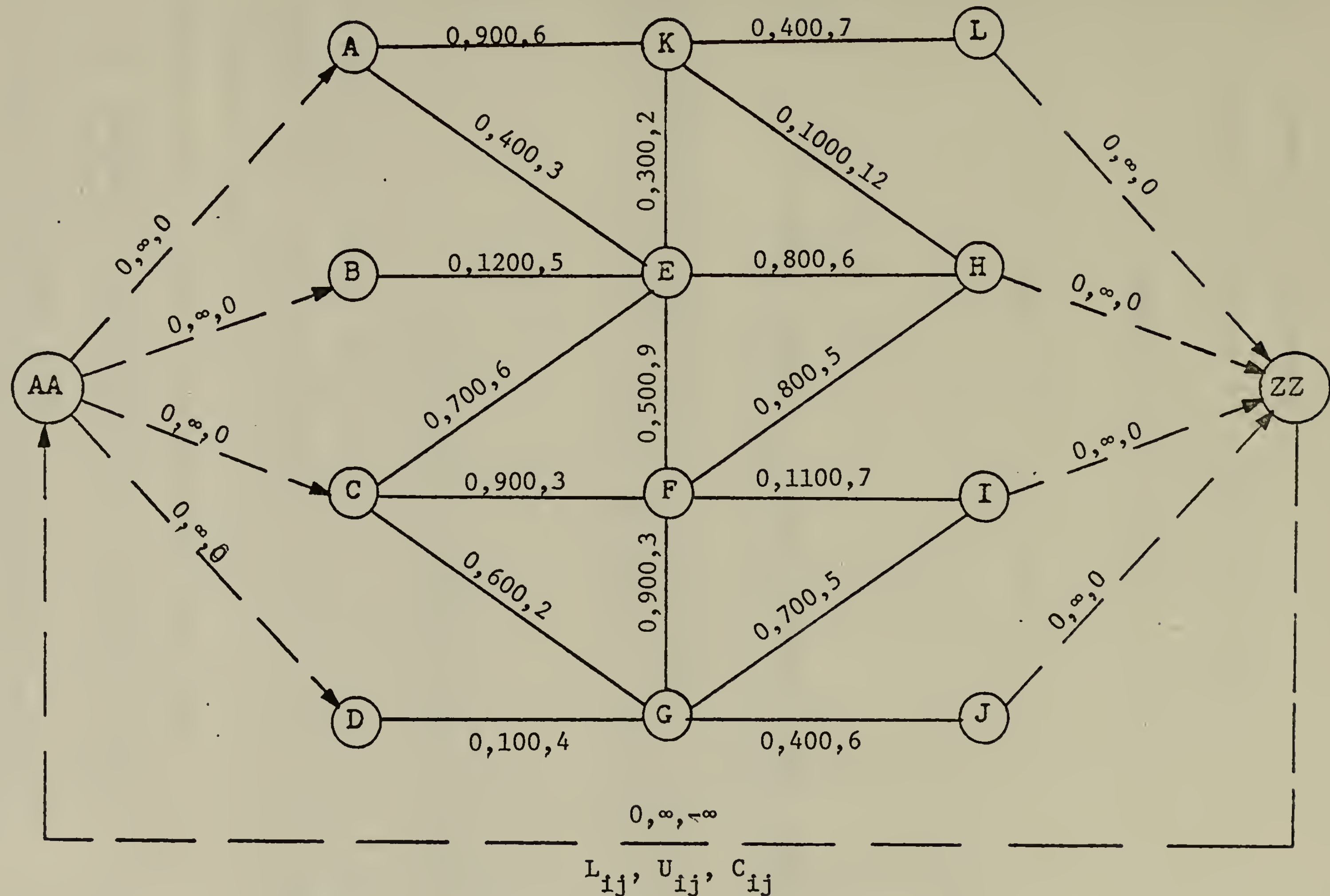


Figure 3. Example Network with Universal Arc

IV. MODEL OPERATION

A. MATHEMATICAL PRELIMINARIES

Assume: (1) The set $X^* = (x_1^*, x_2^*, \ldots, x_n^*)$

maximizes $\sum_{i=1}^{n} f_i(x_i)$

subj. to $\sum_{i=1}^{n} x_i = X \; ; x_i \geq 0$

(2) The $f_i(x_i)$ are continuous and differentiable

(3) $f_i''(x_i) \leq 0$

Note: These are necessary and sufficient conditions for a global maximum since the objective function is concave and the constraints are convex.

Then there exists a scalar $s > 0$ such that:

(1) $f_i'(x_i) = s$ for $x_i^* > 0$

(2) $f_i'(x_i) < s$ for $x_i^* = 0$

Note: The Lagrange Multiplier, s, can be interpreted as the implicit marginal price of allocating an additional unit of resource.

The above is known as Gibbs' Lemma and is discussed in more detail by Danskin [2]. It can be derived from the Kuhn-Tucker conditions or by elementary arguments based on the special structure of the problem. There are two important implications of this lemma:

(1) Marginal return must be above a threshold value, s, for a resource to be allocated

(2) For alternatives which receive allocation, $f_i'(x_i^*) = f_j'(x_j^*)$, i.e., at the optimum allocation point, the ratio (marginal return/marginal cost)$_i$ is constant.

B. MODEL DESCRIPTION

As stated earlier, there are numerous considerations to be weighed in allocating strikes to arcs in the LOC network. From a mathematical viewpoint, it is assumed that the effectiveness of the network to its user may be measured in terms of how to obtain maximum flow at minimum cost. Therefore, the criterion to be used to allocate sorties is to minimize LOC effectiveness over time, taking into account the cost to repair damage on an arc. This is accomplished by allocating strikes to the set of arcs that maximize the costs to the user.

The effect of striking an arc is to reduce its capacity and increase the cost of a minimum cost circulation flow for a specified period of time. Therefore, in allocating N strikes, the selected arcs are the ones that maximize repair cost plus the product of the increase in minimum cost circulation flow and repair time. Specifically, if F is the cost of a minimum cost circulation flow before arc (i,j) is struck, F' is the cost of a minimum cost circulation flow after n_{ij} strikes on arc (i,j), and t_{ij} and k_{ij} are the repair time and repair cost, respectively, for damage caused by n_{ij} strikes, then the arcs selected for attack will be the ones that maximize:

$$\sum_{\text{all } (i,j)} (F' - F)\, t_{ij} + k_{ij}$$

$$\text{s.t.} \quad (1) \quad \sum_{\text{all } (i,j)} n_{ij} = N \;;\; n_{ij} \geq 0$$

where: $$F = \sum_{\text{all } (i,j)} M_{ij} C_{ij}$$

$$F' = \sum_{\text{all }(i,j)} M_{ij}C_{ij} - d_{ij}(C_{ij} + C^u) \quad ; \quad C_{ij} > 0$$

$$C^u << 0$$

Note: C^u is the unit flow cost on the universal arc.

$$d_{ij} = V_{ij}[1 - \exp(-VP_{ij}\ n_{ij})]$$

$$t_{ij} = (d_{ij}/V_{ij})\ T_{ij}$$

$$k_{ij} = (d_{ij}/V_{ij})\ K_{ij}$$

The objective function then becomes to maximize:

$$\sum_{\text{all}(i,j)} -T_{ij}V_{ij}(C_{ij} + C^u)\ [1 - \exp(-VP_{ij}\ n_{ij})]^2 + K_{ij}[1 - \exp(-VP_{ij}\ n_{ij})]$$

This can be shown to be a continuous concave function of n_{ij} whose second derivative is negative. The constraint is convex; therefore, the above objective function becomes the $f_i(x_i)$ of Gibbs' Lemma.

To determine the allocation of N airstrikes, it is necessary to compute the value of the Lagrange Multiplier, s^*. By Gibbs' Lemma, it is known that there exists an $s^* > 0$ such that:

$$n_{ij} = 0 \text{ for } f_i'(x_i) < s^*$$

$$n_{ij} > 0 \text{ for } f_i'(x_i) \geq s^*$$

An iterative method for this computation is developed by Nugent [10].

The above procedure is used in Option (1) where the number of strikes is specified. For Option (2), a similar procedure is developed with the addition of another constraint. Recall that Option (2) requires a percentage by which the initial flow is to

be reduced. The additional constraint becomes:

$$(2) \quad \sum_{\text{all } (i,j)} M_{ij} \geq (1 - p)M'_{ij}$$

where: M'_{ij} is the initial network flow.

p is the percentage by which the initial flow is to be reduced.

Obviously, for the allocation of a maximum of N sorties, either constraint (1) or (2) will be active and the value of the Lagrange Multiplier, s^*, can be computed using Nugent's method.

If the vulnerability parameter of arc (i,j) takes cognizance of the probability of losing an aircraft on any mission against that arc, then the model seeks to interdict that set of arcs where the ratio (marginal return/marginal cost)$_{ij}$ is constant.

C. COMPUTER SOLUTION PROCEDURE AND OUTPUT

The computer solution procedure operates in daily cycles, the strike planner specifying by Option (1) or (2) the specified or maximum number of strikes per day, respectively, and the number of days in the interdiction campaign. The computer solution differs from the optimal model described in B., in that strikes are allocated one at a time. Each time, the arc selected to be struck is the one which maximizes:

$$(F' - F)\, t_{ij} + k_{ij} .$$

The values F', F, t_{ij}, and k_{ij} are computed for one strike instead of for n_{ij} strikes. This is a marginal allocation procedure, the solution having the property that if an arc is selected for interdiction when N strikes are allocated, it will also be selected for

interdiction when more than N strikes are allocated. Since optimal allocation of strikes would not necessarily have this property, the selected arcs are optimal for one strike, but only approximate optimality for multiple strikes.

After each strike, the following information is output:

(1) Arc selected for strike
(2) Arc capacity before and after strike
(3) Actual arc flow before and after strike
(4) Day arc repair is to be completed
(5) Cumulative repair cost
(6) Network flow after strike
*(7) Network flow cost after strike
(8) Number of strikes against that arc for that day

*Note: Since the unit flow cost on the universal arc is an artificial cost used to convert a particular type of source-sink flow problem to one of circulation flow, it is not considered in the printed output calculation.

The end of a campaign day is determined by the option used. Using Option (1), the day terminates when N sorties have been allocated. Using Option (2), the day terminates when either N strikes have been allocated or the total network flow has been reduced to a specified percentage of the initial flow. The following information is output at the end of each day:

(1) Arcs selected for strike
(2) Number of strikes allocated against arc (i,j) for that day
(3) Number of strikes allocated against arc (i,j) for the campaign
(4) Total number of strikes allocated that day
(5) Total number of strikes allocated for the campaign

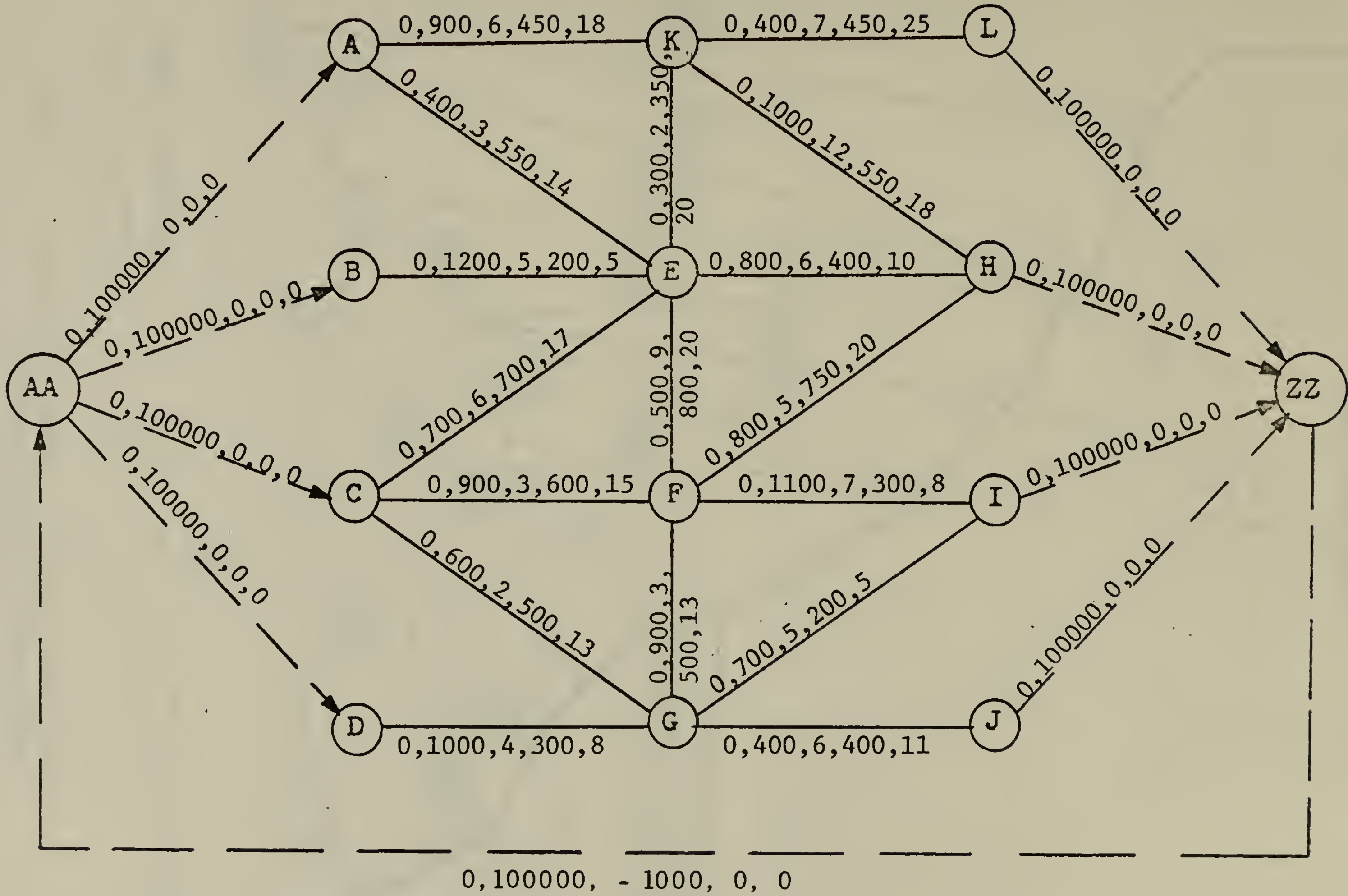
At the start of each day, all arc repair scheduled for that day has been completed. The resulting flow pattern is computed and arc flow, maximum network flow, and network flow cost are output.

V. SAMPLE PROBLEM

The network shown in Fig. 4 is used as a sample interdiction problem. It is the same network used previously but with the addition of maximum repair time and maximum repair cost parameters for each arc. Figure 5 depicts the same network, but node positions in relation to each other have been changed to indicate a more realistic lines-of-communication network. Artificial arcs from the super source are used to indicate that flow in the enemy sanctuary area is invulnerable to attack. Nodes A, B, C, and D are points on supply routes where arc capacity is subject to interdiction. The enemy objective is to move supplies from his sanctuary area to forward supply centers located at nodes L, H, I, and J. Nodes K, E, F, and G are route intersections. Figure 5 also gives the actual direction and magnitude of flow before the interdiction campaign begins.

The objective of the strike planner is to reduce the initial maximum network flow by 60 percent and thereafter, maintain this reduced flow for a campaign of 5 days duration. He has a maximum of 50 aircraft available each day to fly interdiction missions.

The computer solution to this problem is given in Appendix D.



L_{ij}, U_{ij}, C_{ij}, K_{ij}, T_{ij}

Figure 4. Network for Sample Problem

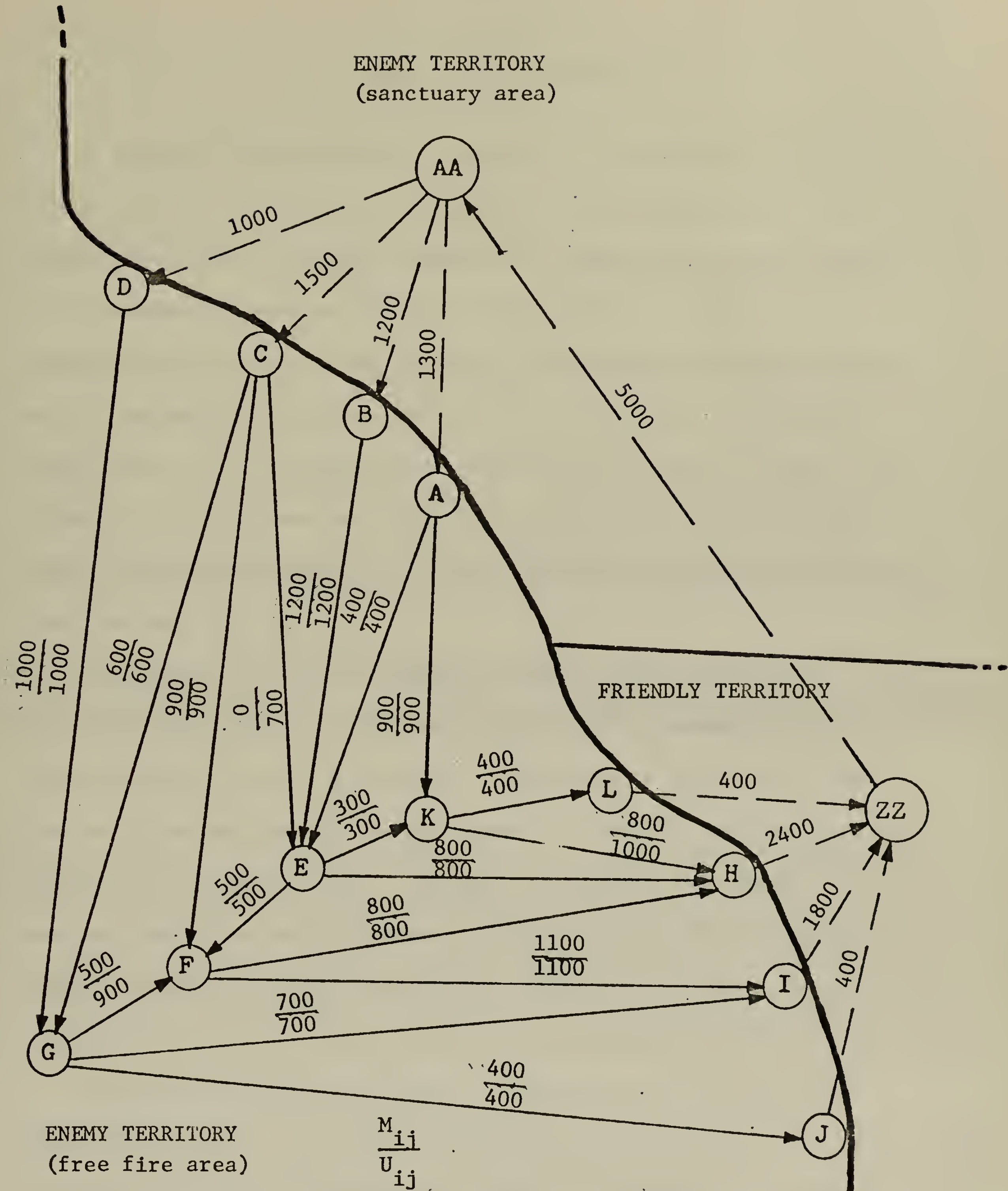


Figure 5. Network for Sample Problem Giving Direction and Magnitude of Initial Flow

VI. SUMMARY AND EXTENSIONS

A computer model has been presented for determining the daily allocation of airstrikes for interdicting a land lines-of-communication (LOC) network, assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is limited by network capacity, and (2) parameter values for each arc are independent. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure operates in daily cycles, performing repair on arcs that have been struck.

Two options are provided that allow the strike planner to (1) specify the number of strikes per day and the number of days in the campaign, or (2) prescribe a percentage by which the initial flow is to be reduced, the maximum number of strikes per day, and the number of campaign days. The upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc in the network are required as input data.

The model has been programmed in FORTRAN IV for use on the IBM 360/67 computer. Detailed information for each day in the campaign is listed in the computer output to include arcs selected for interdiction, network flow and network flow cost at the end of each day, and the number of sorties allocated to selected arcs.

There are almost an endless number of extensions that can be considered in the allocation of airstrikes to a LOC network. In the present model, it is assumed that only one type of aircraft is available to attack the network, i.e., one weapons system against y targets. A more practical and interesting approach is to assume that more than one type of aircraft is available for interdiction, each with a different kill probability against different types of targets, i.e., x weapons systems against y targets. The damage on an arc caused by n_{ij} strikes then becomes:

$$d_{ij} = V_{ij} \left[1 - \prod_{k=1}^{x} \exp(-VP_{ij,k}\, n_{ij,k})\right]$$

Another modification would be to vary the damage function itself. The exponential was used in this model because it exhibits decreasing marginal returns to scale and simplifies computational procedure; however, other such functions could be utilized. The exponential is a continuous approximation, for n_{ij} large and VP_{ij} small, of the function:

$$V_{ij}[1 - (1 - VP_{ij})^{n_{ij}}]$$

A contrast could be made of the damage caused by both distributions with actual damage data. Holliday [5] has done research in the field of damage assessment for the Advanced Research Projects Agency and his work might be used for such a contrast.

APPENDIX A

COMPUTER PROGRAM INPUT DATA

Card no.	Column no.	Format	Description
1	1-80	A	Program title. (PRONAM)
2	1-5	I5	Maximum number of strikes that can be allocated each day. (N)
	6-10	I5	The number of days in the campaign. (NUMDAY)
	11-15	I5	The number of invulnerable or artificial arcs in the network. Include all arcs considered invulnerable to airstrike, artificial arcs connecting the dummy source to the true source(s), and artificial arcs connecting the true sink(s) to the dummy sink. (INVL)
	16-20	I5	If equal to 1, specifies one-way flow on some arcs in the network. If equal to 2, specifies two-way flow on all arcs in the network. When arcs are two-way, a strike against one implies a strike against the other. (X2WAY)
	22-26	F4.2	Specifies the percentage of flow to be interdicted. If greater than 1.0, a fixed number of strikes per day (N) will be targeted. (REDU)
Arc Data Cards	1-4	A4	Name of arc. If two-way flow, must be the same for both arcs. (ARNAM)
	6-7	A2	Beginning node. (FROM)
	9-10	A2	Ending node. (TO)
	12-20	I9	Lower bound on arc flow. (LBND)
	22-30	I9	Arc capacity (upper bound on arc flow). (MCTY)
	32-40	I9	Cost of one unit of flow on the arc. (JFCST)

	42-50	I9	Maximum arc repair cost, i.e., the cost of repairing the arc from its lower bound capacity to its maximum capacity. (RMAXCT)
	52-60	I9	Maximum arc repair time, i.e., the time required to repair the arc from its lower bound capacity to its maximum capacity. (MRTM)
	66-70	F5.3	Arc vulnerability parameter. (VP)
Last Card	1-2	A4	The word 'GO'.

Note: Arc data cards must be placed in the following order:

1. Universal arc.
2. Invulnerable and artifical arcs.
3. Vulnerable arcs.

APPENDIX B

GLOSSARY OF PROGRAM TERMS

ARNAM(k)	The designation of arc ij. (input data)
COSTMN(k)	Real number equivalent of MNCOST.
CRC	Cumulative repair cost for all interdicted arcs.
CRT(k)	Cumulative repair time for an interdicted arc.
CSTMNT	Real number equivalent of MNTCST.
DAMAGE(k)	Total reduction in capacity of arc ij caused by one or more strikes against it.
DAY	Counter for number of days up to NUMDAY.
FLOW(k)	Amount of flow on arc ij.
FROM(k)	Location of beginning node of arc ij. (input data)
GO	Flag used to denote last data card has been read.
IKEEP1	Used to save capacity of arc ij.
IKPOP2	Used to save capacity of the optimal arc to strike.
IKPOP1	If OPTARC is two-way, saves the capacity of the reverse direction arc.
IMNCST	Cost of the initial flow in the network.
IMNCUT	Cost of flow in the network after the optimal arc has been struck.
INFEAS	A flag used in the Out-of-Kilter subroutine to denote infeasible flow.
INVL	The number of invulnerable or artificial arcs in the network. (input data)
IOPARC	If an arc is two-way, specifies the reverse direction of the best arc to strike.
IRQFLO	Integer equivalent of REQFLO.

IS(k)	Counter for arcs that have been struck one or more times.
ISWTCH	Flag used to denote a two-way arc.
IT(k)	Counter for total strikes on arc ij during the campaign.
IV(k)	Counter for total strikes on arc ij during one day of the campaign.
JFCST(k)	Cost of one unit of flow on arc ij. (input data)
JFLOW(k)	Saves flow on OPTARC before strike. (for output)
J2WAY	Same as ISWTCH.
KFLOI	Saves flow on OPTARC after strike. (for output)
LBND(k)	Lower flow bound on arc ij. (input data)
LINE	Line counter used in the output subroutine.
MCAP(k)	Capacity of arc ij before next strike.
MCAPU	Saves the capacity of OPTARC before strike.
MCTY(k)	Flow capacity of arc ij. (input data)
MNCOST	Network flow cost before next strike.
MNTCST	Network flow cost after arc ij has been struck.
MRTM(k)	Maximum repair time for arc ij. (input data)
N	Maximum number of strikes that can be allocated during any one day. (input data)
NEWCAP(k)	Capacity of arc ij after strike.
NN(k)	Unique node number assigned by Subroutine Number.
NR	Counter for total strikes on all arcs during any one day.
NRC	Counter for total campaign strikes.
NUMARC	Maximum number of arcs in the network.
NUMDAY	Number of days in the campaign. (input data)

NUMNDE	Maximum number of nodes in the network.
OPTARC	The optimal arc to strike.
REDU	The percentage by which initial flow is to be reduced. (input data)
REPDAY(k)	Day on which arc ij is to be repaired.
RMAXCT(k)	Maximum repair cost for arc ij. (input data)
RPMCST(k)	Marginal repair cost for arc ij.
TME(k)	Marginal repair time for arc ij.
TO(k)	Location of ending node of arc ij. (input data)
TOTCST	Counter for cumulative campaign strikes.
TOTDST	Counter for cumulative day strikes.
VALUE(k)	The value of striking arc ij.
VP(k)	Vulnerability parameter of arc ij. (input data)
VULCAP(k)	The vulnerable capacity of arc ij.
XINCRE(k)	Marginal reduction in capacity on arc ij caused by an additional strike.
X2WAY	Input parameter that specifies whether or not all arcs are two-way.

APPENDIX C

PROGRAM LISTING

```
      DIMENSION XINCRE(150)
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLCI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      COMMON/DAT14/TME(150),VULCTY(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC
      DATA NAME/'GO'/

      OPTFLO = 0.
      REQFLO = 0.
      IRQFLO = 0
      I=0
C     READ PROGRAM TITLE AND PROGRAM PARAMETERS
      READ(5,505)(PRONAM(KK),KK=1,20)
      READ(5,500)N,NUMDAY,INVL,X2WAY,REDU
      WRITE(6,504)
      S = INVL + 2
C     READ ARC PARAMETERS
   10 I=I+1
      READ(5,510)ARNAM(I),FROM(I),TO(I),LBND(I),MCTY(I)
     1,JFCST(I),RMAXCT(I),MRTM(I),VP(I)
      IF(ARNAM(I) .EQ. NAME) GO TO 20
C     DETERMINE PARAMETERS FOR FIRST STRIKE
      VULCAP(I) = MCTY(I) - LBND(I)
      MCAP(I) = MCTY(I)
      DAMAGE(I) = VULCAP(I)*(1.0-EXP(-VP(I)))
      NEWCP(I) = FLOAT(MCTY(I))-DAMAGE(I) + .5
      RPMCST(I) = (FLOAT(RMAXCT(I))/VULCAP(I))*DAMAGE(I)
      TME(I) = (FLOAT(MRTM(I))/VULCAP(I))*DAMAGE(I)
      GO TO 10
   20 CONTINUE
      NUMARC = I-1
C     CALL NUMBER SUBROUTINE TO DESCRIBE NETWORK
      CALL NUMBER
C     INITIALIZE VARIABLES
      DO 30 K = 1,NUMARC
      IS(K) = 0
      IT(K) = 0
      IV(K) = 0
      FLOW(K) = 0
      NL(K) = 0
```

```
      REPDAY(K) = 0
      TOTCST(K) = 0
      TOTDST(K) = 0
      CRT(K) = 0.
      VALUE(K) =0.
      VULCTY(K) = 0.
      XINCRE(K) = 0.
   30 CONTINUE
C     SOLVE UNINTERDICTED NETWORK
      CALL OCK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1) GO TO 70
      OPTFLO = FLOAT(FLOW(1))
      REQFLO = OPTFLO*(1.-REDU)
      IRQFLO = REQFLO
C     PRINT PROGRAM TITLE AND MODE OF OPERATION
      CALL OUTPUT(1)
      DAY = 0
      CALL OUTPUT(6)
C     FIND MAX FLOW,FLOW COST,AND PRINT RESULTS
      CALL FLOCST(FLOW,JFCST,NUMARC,IMNCST,INVL,1)
      CALL OUTPUT(7)
      DAY = 1
      NR = 0
      NRC = 0
      CRC = 0
C     INTERDICTION OF NETWORK BEGINS HERE
   40 NR = NR + 1
      NRC = NRC + 1
      INFEAS = 0
      DO 45 I=S,NUMARC
      JFLOW(I) = FLOW(I)
   45 CONTINUE
C     CALL STKARC TO DETERMINE BEST ARC TO STRIKE
      CALL STKARC
      IF(INFEAS.EQ.1) GO TO 70
C     OPTARC IS THE BEST ARC
      MCAPU = MCAP(OPTARC)
      KFLOI = FLOW(OPTARC)
      ISAVE = OPTARC
      K = IOPARC
C     STORE COST OF REPAIR AND COST OF STRIKE
      CRC = CRC + RPMCST(OPTARC) + .5
   50 CONTINUE
      CRT(K) = CRT(K) + TME(K)
      REPDAY(K) = DAY + CRT(K) + .5
C     COUNTER FOR ARCS THAT HAVE BEEN STRUCK
      IS(K) = IS(K) + 1
C     COUNTER FOR DAY STRIKES
      IV(K) = IV(K) + 1
C     COUNTER FOR CAMPAIGN STRIKES
      IT(K) = IT(K) + 1
```

```
      TOTDST(K) =  IV(K)
      TOTCST(K) = IT(K)

C     DETERMINE PARAMETERS FOR NEXT STRIKE

      J = IS(K) + 1
      MCAP(K) = NEWCP(K)
      VULCTY(K)=MCAP(K) - LBND(K)
      XINCRE(K)=VULCTY(K)*(1.-EXP(-VP(K)))
      NEWCP(K)=FLOAT(MCAP(K))-XINCRE(K) + .5
      DAMAGE(K) = MCTY(K) - NEWCP(K)
      RPMCST(K) = (FLOAT(RMAXCT(K))/VULCAP(K))*XINCRE(K)
      TME(K) =(FLOAT(MRTM(K))/VULCAP(K))*XINCRE(K)

C     IF ARC IS 2-WAY, GO BACK AND COMPUTE
C     PARAMETERS FOR RETURN ARC

      IF (ISWTCH.EQ.1) GO TO 60
      K = ISAVE
      ISWTCH = 1
      GO TO 50
   60 CONTINUE

C     PRINT STRIKE RESULTS

      CALL OUTPUT(2)

   70 CONTINUE
      IF(INFEAS .EQ. 1) GO TO 130
      IF(FLOW(1) .LT. LBND(1)) GO TO 130
      IF(FLOW(1) .EQ. 0) GO TO 140

C     IF REDU IS < 1,NUMBER OF STRIKES IS NOT
C     CONSTANT. IF NUMBER OF STRIKES = NUMBER
C     SPECIFIED IN INPUT OR IF CAPACITY
C     HAS BEEN REDUCED TO SPECIFIED AMOUNT,
C     DAY IS DONE.

      IF(REDU.LE.1.0)GO TO 80
      IF(NR .EQ. N) GO TO 90
      GO TO 40
   80 CONTINUE
      IF(FLOW(1).LE.REQFLO)GO TO 90
      IF(NR.EQ.N)GO TO 90
      GO TO 40
   90 CALL OUTPUT(5)
      IF(DAY .EQ. NUMDAY) GO TO 150
      LINE = 48

C     RE-INITIALIZE PARAMETERS FOR NEXT DAY

  100 DAY = DAY + 1
      NR =0
      DO 110 M=S,NUMARC
      IV(M) = 0
      TOTDST(M) = 0
  110 CONTINUE

C     PERFORM REPAIR ON ARCS

      DO 120 M = S,NUMARC
      IF(IS(M) .EQ. 0) GO TO 120
      MCAP(M)=FLOAT(MCAP(M))+(1./CRT(M))*DAMAGE(M)
      VULCTY(M)=MCAP(M) - LBND(M)
      XINCRE(M)=VULCTY(M)*(1.-EXP(-VP(M)))
      NEWCP(M)=FLOAT(MCAP(M)) - XINCRE(M) + .5
      DAMAGE(M)=MCTY(M) - NEWCP(M)
      RPMCST(M) = (FLOAT(RMAXCT(M))/VULCAP(M))*XINCRE(M)
      TME(M)=(FLOAT(MRTM(M))/VULCAP(M))*XINCRE(M)
      CRT(M) = CRT(M) - 1.
  120 CONTINUE
```

```
C       SOLVE NETWORK FOR NEXT DAY

        CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
       1PI,NL,INFEAS)
        IF(INFEAS .EQ. 1) GO TO 70

C       FIND  FLOW COST

        CALL FLOCST(FLOW,JFCST,NUMARC,IMNCST,INVL,1)

C       PRINT NETWORK FLOW PATTERN AND ITS COST

        CALL OUTPUT(6)
        CALL OUTPUT(7)
        IF(FLOW(1).GT.REQFLO)GO TO 40
        GO TO 100
   130  CALL OUTPUT(3)
        GO TO 90
   140  CALL OUTPUT(4)
        GO TO 90
   150  CONTINUE
        CALL EXIT


   500  FORMAT(4I5,1X,F4.2)
   504  FORMAT(1H1,10X)
   505  FORMAT(20A4)
   510  FORMAT(A4,2(1X,A2),5(1X,I9),5X,F5.3)
        END
```

```
C     OOK IS THE FULKERSON OUT OF KILTER ALGORITHM

      SUBROUTINE OOK(NODES,ARCS,I,J,COST,HI,LO,FLOW,PI
     1,NA,INFEAS)
      DIMENSION I(150),J(150),COST(150),HI(150),
     1LO(150),FLOW(150)
      DIMENSION PI(150),NA(150)
      INTEGER NODES,ARCS,I,J,COST,HI,LO,FLOW,PI,NA,INFEAS
      INTEGER A,AA,N,SRC,SNK,DEL,INF,C,AOK,COK,N1,N2,
     1INC,LABEL
C         DEFINITION OF CALLING SEQUENCE
C
C         NAME        USE
C
C         NODES       NUMBER OF NODES
C         ARCS        NUMBER OF ARCS
C         I           LIST OF FROM (BEGINNING) NODES
C         J           LIST OF TO (ENDING) NODES
C         COST        UNIT COST OF FLOW ON ARCS
C         HI          UPPER BOUNDS FOR ARCS
C         LO          LOWER BOUNDS FOR ARCS
C         FLOW        AMOUNT OF FLOW IN ARCS
C         PI          NODE PRICES
C         NA          SCRATCH LIST FOR NODES LABELING
C         INFEAS      FLAG DENOTING THE CONDITION OF OUTPUT
      INF = 2147483647
      AOK = 0
C     LOOK FOR AN OUT OF KILTER ARC
      AA=1
  100 N1 = I(AA)
      N2 = J(AA)
      C = COST(AA) + PI(N1) - PI(N2)
   40 IF(FLOW(AA).LT.LO(AA).OR.(C.LT.0.AND.FLOW(AA).LT.
     1HI(AA))) GO TO 50
      IF(FLOW(AA).GT.HI(AA).OR.(C.GT.0.AND.FLOW(AA).GT.
     1LO(AA))) GO TO 60
   90 AA = AA + 1
      IF(AA.LE.ARCS)GO TO 100
C     NO OUT OF KILTER ARCS LEFT
      INFEAS = 0
      RETURN
C     OUT OF KILTER ARC FOUND
   50 SRC = J(AA)
      SNK = I(AA)
      LABEL= +AA
      GO TO 200
   60 SRC = I(AA)
      SNK = J(AA)
      LABEL = -AA
C     SAVE LABELS IF LAST OPERATION WAS INCREASING NODE
C     PRICES ON THIS ARC
  200 IF(AA.EQ.AOK.AND.NA(SRC).NE.0) GO TO 205
      DO 201 N =1,NODES
      NA(N) = 0
  201 CONTINUE
      AOK = AA
  205 COK = C
      NA(SRC) = LABEL
```

```
C     LABEL

  210 LABEL = 0
      DO 250 A =1,ARCS
      N1 = I(A)
      IF(N1.LT.0) GO TO 250
      N2 = J(A)
      IF(NA(N1).EQ.0.AND.NA(N2).EQ.0) GO TO 250
      IF(NA(N1).NE.0.AND.NA(N2).NE.0)GO TO 245
      C = COST(A) + PI(N1) -PI(N2)
      IF(NA(N1).EQ.0) GO TO 220
      IF(FLOW(A).GE.HI(A).OR.(FLOW(A).GE.LO(A).AND.C.GT.0))
     1GO TO 245
      NA(N2) = A
      GO TO 240
  220 IF(FLOW(A).LE.LO(A).OR.(FLOW(A).LE.HI(A).AND.C.LT.0))
     1GO TO 245
      NA(N1) = -A
  240 LABEL = 1

C     NODE LABELLED, TEST FOR BREAKTHRU

      IF(NA(SNK).NE.0) GO TO 260
  245 I(A) = -N1
  250 CONTINUE

C     GO BACK AND DO MORE LABELLING IF SOME NODE WAS
C     LABELLED ON LAST PASS

      IF(LABEL.NE.0) GO TO 210

C     RESTORE POSITIVE SIGNS TO FIRST NODE LIST

  260 DO 270 A=1,ARCS
      I(A) = IABS(I(A))
  270 CONTINUE

C     IF NO LABELLING DONE ON LAST PASS, GO TO
C     INCREASE PIE

      IF(LABEL.EQ.0) GO TO 400

C     BREAKTHRU,  FIND THE INCREMENT

  300 INC = INF

C     FOLLOW PATH BACK FROM SOURCE

      N = SRC
  310 A = IABS(NA(N))
      IF(NA(N).LT.0) GO TO 315
      N2 = I(A)
      C = COST(A) - PI(N) + PI(N2)
      IF(C.GT.0)INC = MIN0(INC,LO(A) - FLOW(A))
      IF(C.LE.0)INC = MIN0(INC,HI(A) -FLOW(A))
      GO TO 340
  315 N2 = J(A)
      C = COST(A) + PI(N) -PI(N2)
  320 IF(C.LT.0)INC = MIN0(INC,FLOW(A) -HI(A))
      IF(C.GE.0)INC = MIN0(INC,FLOW(A) -LO(A))
  340 N = N2
      IF(N.NE.SRC)GO TO 310

C     INCREMENTS

  350 A = IABS(NA(N))
      IF(NA(N).LT.0)GO TO 360
      FLOW(A) = FLOW(A) + INC
      N = I(A)
      GO TO 370
  360 FLOW(A) = FLOW(A) -INC
      N = J(A)
```

```
  37C IF(N.NE.SRC)GO TO 350
C       FLOW INCREMENTED, RETURN TO KILTER TEST
        NA(N) = 0
        GO TO 100
C       CHANGE PI
  400 DEL = INF
C       FIND INCREMENT
        DO 420 A=1,ARCS
        N1 = I(A)
        N2 = J(A)
        IF(NA(N1).EQ.0.AND.NA(N2).EQ.0)GO TO 420
        IF(NA(N1).NE.0.AND.NA(N2).NE.0)GO TO 420
        C = COST(A) + PI(N1) - PI(N2)
        IF(NA(N2).EQ.0.AND.FLOW(A).LT.HI(A)) DEL =MIN0(DEL,C)
        IF(NA(N2).NE.0.AND.FLOW(A).GT.LO(A)) DEL =MIN0(DEL,-C)
  420 CONTINUE
        IF(DEL.NE.INF) GO TO 430
        IF(FLOW(AA).EQ.LO(AA).OR.FLOW(AA).EQ.HI(AA))GO TO 425
C       INFEASIBLE SOLUTION
        INFEAS = 1
        RETURN
C       INCREASE PI
  425 DEL = IABS(COK)
  430 DO 450 N =1,NODES
        IF(NA(N).EQ.0) PI(N) = PI(N) + DEL
  450 CONTINUE
C       GO BACK TO KILTER TEST
        GO TO 100
        END
```

```
      SUBROUTINE FLOCST(FLOW,COST,NUMARC,MINCT,INVL,JUNIV)
      DIMENSION FLOW(150),COST(150)
      INTEGER FLOW,COST

      MINCT = 0
      IUNV = INVL

C     IF JUNIV = 0, USE COST OF UNIVERSAL ARC

      IF(JUNIV .EQ. 0) IUNV = 1
      DO 10 I = IUNV,NUMARC
   10 MINCT = MINCT + FLOW(I)*COST(I)
      RETURN
      END



      SUBROUTINE NUMBER
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO

C     NUMBER THE NODES

      NUMNDE = 0
      DO 30 L=1,NUMARC
      NN(NUMNDE + 1) =FROM(L)
      FROM(L) = NODNUM(FROM(L),NN)
      NUMNDE =MAX0(NUMNDE,FROM(L))
   30 CONTINUE
      DO 40 L =1,NUMARC
      NN(NUMNDE + 1) = TO(L)
      TO(L) = NODNUM(TO(L),NN)
      NUMNDE = MAX0(NUMNDE,TO(L))
   40 CONTINUE

C     CHECK FOR DEAD NODES

      DO 48 I =1,NUMNDE
      DO 42 L =1,NUMARC
      IF(FROM(L).EQ.1)GO TO 44
   42 CONTINUE
      WRITE(6,522)NN(I)
   44 CONTINUE
      DO 46 L =1,NUMARC
      IF(TO(L).EQ.1)GO TO 48
   46 CONTINUE
      WRITE(6,530)NN(I)
   48 CONTINUE
      RETURN

  522 FORMAT(1H1, 22HNO ARC BEGINS AT NODE , A4/1H1)
  530 FORMAT(1H1, 22HNO ARC ENDS AT NODE   , A4/1H1)
      END



      FUNCTION NODNUM(III,NN)
      DIMENSION NN(1)
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N

      NODNUM = NUMNDE + 1
      IF(NUMNDE.EQ.0)RETURN
      DO 10 I=1,NUMNDE
      IF(III.EQ.NN(I)) NODNUM = I
   10 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE STKARC
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLCI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      COMMON/DAT14/TME(150),VULCTY(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC

C     INITIALIZE PARAMETERS

      MNTCST = 0
      MNCOST = 0
      CSTMNT=0.
      COSTMN=0.
      UPVAL = -10000000.

C     SOLVE FOR LAST FLOW PATTERN

      CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,MNCOST,INVL,0)
      DO 100 I=S,NUMARC
      IF(FLOW(I).EQ.0)GO TO 100
      IKEEP1=MCAP(I)
      MCAP(I)=NEWCP(I)

C     SOLVE INTERDICTED FLOW PATTERN FOR
C     SELECTED ARCS AND FIND FLOW COST

      CALL OOK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,MNTCST,INVL,0)

C     FIND VALUE OF STRIKING ARCS AND
C     SAVE MAXIMUM VALUE

      MCAP(I)=IKEEP1
      CSTMNT=MNTCST
      COSTMN=MNCOST
      VALUE(I)=(CSTMNT-COSTMN)*TME(I)+RPMCST(I)
      IF(VALUE(I).LT.UPVAL)GO TO 90
      UPVAL=VALUE(I)

C     OPTARC IS THE BEST ARC TO STRIKE

      OPTARC=I
   90 CONTINUE
  100 CONTINUE
      DO 52 I=S,NUMARC
      IF(FROM(OPTARC).EQ.TO(I).AND.TO(OPTARC).EQ.FROM(I)
     1.AND.ARNAM(OPTARC).EQ.ARNAM(I))GO TO 54
   52 CONTINUE
   53 CONTINUE
      J2WY = 1

C     IOPARC IS THE REVERSE DIRECTION ARC

      IOPARC = OPTARC
```

```
      GO TO 56
   54 J2WY = 2
      IOPARC = I
      ISWTCH = J2WY
      IF(X2WAY.EQ.1) GO TO 53
   56 CONTINUE
      IKPOP2 = MCAP(IOPARC)
      IKPOP1 = MCAP(OPTARC)
      MCAP(OPTARC) = NEWCP(OPTARC)
      MCAP(IOPARC) = NEWCP(IOPARC)
C
C     STRIKE BEST ARC AND FIND FLOW COST
C
      CALL OCK(NUMNDE,NUMARC,FROM,TO,JFCST,MCAP,LBND,FLOW,
     1PI,NL,INFEAS)
      IF(INFEAS.EQ.1)RETURN
      CALL FLOCST(FLOW,JFCST,NUMARC,IMNCUT,INVL,1)
      MCAP(OPTARC) = IKPOP1
      MCAP(IOPARC) = IKPOP2
      RETURN
      END
```

```
      SUBROUTINE OUTPUT(J)
      COMMON/DAT01/INVL,DAY,NUMDAY,NUMARC,OPTARC,LINE,INFEAS
      COMMON/DAT02/S,NUMNDE,MCAPU,KFLOI,NR,N
      COMMON/DAT03/NRC,IMNCST,IMNCUT,X2WAY,MNCOST,MNTCST
      COMMON/DAT04/REPDAY(150),ARNAM(150),FROM(150),TO(150)
      COMMON/DAT05/RMAXCT(150),RPMCST(150),PI(150)
      COMMON/DAT06/LBND(150),NEWCP(150),MCAP(150),IS(150)
      COMMON/DAT07/MRTM(150),NN(150),MCTY(150),JFCST(150)
      COMMON/DAT08/FLOW(150),NL(150)
      COMMON/DAT09/VALUE(150),PRONAM(25),JFLOW(150)
      COMMON/DAT10/REQFLO,REDU,J2WY,ISWTCH,CRC
      COMMON/DAT11/IOPARC,TOTDST(150),TOTCST(150),IT(150)
      COMMON/DAT12/IV(150),DAMAGE(150),VP(150)
      COMMON/DAT13/IRQFLO,VULCAP(150),CRT(150)
      INTEGER TOTDST,TOTCST,DAY,ARNAM,X2WAY,FROM,TO
      INTEGER FLOW,PI,OPTARC,REPDAY,S,RMAXCT,CRC

      IF(J.GT.1) GO TO  20
      WRITE(6,506)(PRONAM(I),I=1,20)
      IF(REDU.LT.1.)GO TO 10
      WRITE(6,507)NUMDAY,N
      LINE = 8
      RETURN
   10 WRITE(6,510)N,NUMDAY,FLOW(1),IRQFLO,IRQFLO,N
      WRITE(6,504)
      LINE = 0
      RETURN
   20 IF(LINE.NE.48) GO TO  30
      WRITE(6,504)
      LINE = 0
   30 IF(LINE.NE.0)GO TO  40
   40 GO TO(100,100,150,160,210,50,180),J
   50 IARC = 0
      IF(DAY.EQ.0)WRITE(6,502)
      IF(DAY.EQ.0)GO TO 70
      WRITE(6,519)DAY
   60 LINE = 0
   70 WRITE(6,508)
   80 IARC = IARC + 1
      IF(IARC.GT.NUMARC) GO TO 90
      IF(FLOW(IARC).EQ.0. AND.DAY.NE.0) GO TO 80
      LINE = LINE + 1
      II = FROM(IARC)
      IJ = TO(IARC)
      WRITE(6,509)ARNAM(IARC),NN(II),NN(IJ),LBND(IARC),
     1MCAP(IARC),JFCST(IARC),RMAXCT(IARC),MRTM(IARC),
     2FLOW(IARC),VP(IARC)
      IF(LINE.NE.49) GO TO 80
      WRITE(6,504)
      GO TO 60
   90 CONTINUE
      LINE = 0
      RETURN
  100 CONTINUE
      IF(LINE +6.GE.48) GO TO 200
      IF(LINE.NE.0) GO TO 120
  110 CONTINUE
      WRITE(6,503)DAY
      WRITE(6,500)
      LINE = LINE + 4
  120 CONTINUE
      K = OPTARC
      III=FROM(K)
      JJJ=TO(K)
      WRITE(6,501)NR,NN(III),NN(JJJ),MCAPU,MCAP(K),
     1JFLOW(K),KFLOI,REPDAY(K),CRC,FLOW(1),IMNCUT
     2,IS(K)
  130 LINE = LINE + 1
      RETURN
  150 WRITE(6,513)DAY
```

```
      LINE =LINE + 8
      RETURN
  160 WRITE(6,512)DAY
      LINE = LINE + 8
      RETURN
  180 WRITE(6,515) IMNCST,FLOW(1)
      IF(DAY.EQ.0)GO TO 190
      IF(FLOW(1).GT.REQFLO)GO TO 190
      WRITE(6,516)DAY
  190 CONTINUE
      WRITE(6,504)
      LINE = 0
      RETURN
  200 LINE =48
      GO TO  20
  210 CONTINUE
      WRITE(6,520)DAY
      WRITE(6,521)DAY
      DO 220 JARC=S,NUMARC
      KK = FROM(JARC)
      KJ = TO(JARC)
      IF(TOTDST(JARC).EQ.0.AND.TOTCST(JARC).EQ.0)
     1 GO TO 220
      IF(FLOW(JARC).EQ.0)GO TO 220
      WRITE(6,522)ARNAM(JARC),NN(KK),NN(KJ),TOTDST(JARC),
     1TOTCST(JARC)
  220 CONTINUE
      WRITE(6,523)NR,NRC
      RETURN

  500 FORMAT(T30,'ARC CAPACITY',T48,'ARC FLOW',T61,'TO BE',
     1T71,'CUM.',T77,'MAXIMUM',T87,'NETWORK',T96,'NUMBER',/
     2,T10,'MISSION',T28,'BEFORE  AFTER',4X,'BEFORE  AFTER'
     3,2X,'REPAIRED',2X,'REPAIR',1X,'NETWORK',4X,'FLOW',4X,
     4'OF ARC',/,T10,'NUMBER',2X,'FROM  TO',2X,'STRIKE',2X,
     5'STRIKE',3X,'STRIKE  STRIKE',2X,'ON DAY',4X,'COST',3X,
     6'FLOW',6X,'COST',4X,'STRIKES')

  501 FORMAT(10X,I3,5X,A2,3X,A2,2X,2(I5,3X),1X,2(I5,3X),
     1I4,5X,I5,1X,I7,1X,I9,3X,I4)

  502 FORMAT('0',19X,'INITIAL FLOW PATTERN FOR LOC NETWORK'
     1,/)

  503 FORMAT('0',8X,'STRIKES TARGETED FOR DAY',I3,'.')

  504 FORMAT(1H1,10X)

  506 FORMAT(1H ,28X,20A4,///)

  507 FORMAT(1H ,32X, 23HCAMPAIGN WILL  LAST FOR   ,I3,
     112H DAYS, WITH  ,I3,' MISSIONS PER DAY.')

  508 FORMAT(44X,'LOWER',6X,'UPPER',13X,'MAX.',5X,'MAX.',15X
     1,'ARC',/,20X,'ARC',4X,'FROM',5X,'TO',6X,'FLOW',7X,
     2'FLOW',5X,'FLOW',4X,'REPAIR',3X,'REPAIR',4X,'ARC',7X,
     3'VULNER.',/,20X,'NAME',3X,'NODE',4X,'NODE',5X,'BOUND',
     46X,'BOUND',4X,'COST',5X,'COST',5X,'TIME',5X,'FLOW',
     56X,'PARAM.')

  509 FORMAT(2H  ,18X,A4,4X,A2,6X,A2,1X,I8,5X,I8,1X,I7,
     12(4X,I5),1X,I8,6X,F5.3)

  510 FORMAT(36X,'MAXIMUM NUMBER OF STRIKES EACH DAY :'
     1,I3,/,36X,'NUMBER OF DAYS IN THE CAMPAIGN     :',I3,
     2//,36X,'NOTE: MISSIONS WILL CONTINUE EACH DAY UNTIL',
     3/,36X,'ONE OF THE FOLLOWING OCCURS :',/,36X,
     4'(1) UNINTERDICTED FLOW (',I4,' UNITS) IS',/,40X,
     5'REDUCED TO ',I4,' UNITS.',/,36X,'(2) FLOW AT THE ',
     6'BEGINNING OF EACH',/,40X,'DAY IS REDUCED TO ',I4,
     7' UNITS.',/,36X,'(3) ',I3,' STRIKES HAVE BEEN '
     8,'TARGETED.')
```

```
512 FORMAT(1H ,///10X,27HFLOW TOTALLY STOPPED ON DAY   ,
   1I10,///)
513 FORMAT(1H ,///10X,30HMINIMUM THRU-PUT CANNOT BE MET,
   1' ON DAY',I10,///)
515 FORMAT('0',//,20X,'FLOW COST  :',I8,/,20X,
   1'THROUGHPUT :',I8)
516 FORMAT('0',//,20X,'NO STRIKES REQUIRED ON DAY',I3,'.')
519 FORMAT('0',19X,'ALL ARC REPAIR SCHEDULED FOR DAY',I3,
   1' HAS BEEN COMPLETED. ',/,20X,'RESULTING FLOW PATTERN'
   2,' IS AS FOLLOWS:',/)
520 FORMAT('0',/,20X,'RECAP FOR DAY',I3,':',/,20X,
   1'*****************')
521 FORMAT('0',45X,'TOTAL NUMBER    TOTAL NUMBER',/,21X,
   1' ARC',21X,'ARC STRIKES     ARC STRIKES',/,22X,'NAME',
   24X,'FROM',5X,'TO',5X,'FOR DAY',I3,6X,'FOR CAMPAIGN')
522 FORMAT(2H  ,20X,A4,5X,A2,6X,A2,9X,I4,12X,I4)
523 FORMAT('0',/,22X,'NUMBER STRIKES THIS DAY      :',
   1I9,/,22X,'NUMBER STRIKES THIS CAMPAIGN :',I9)
    END
```

APPENDIX D

COMPUTER OUTPUT FOR SAMPLE PROBLEM

INTERDICTION OF A LINES OF COMMUNICATION NETWORK

```
MAXIMUM NUMBER OF STRIKES EACH DAY : 50
NUMBER OF DAYS IN THE CAMPAIGN     :  5

NOTE: MISSIONS WILL CONTINUE EACH DAY UNTIL
ONE OF THE FOLLOWING OCCURS :
(1) UNINTERDICTED FLOW (5000 UNITS) IS
    REDUCED TO 2000 UNITS.
(2) FLOW AT THE BEGINNING OF EACH
    DAY IS REDUCED TO 2000 UNITS.
(3)  50 STRIKES HAVE BEEN TARGETED.
```

INITIAL FLOW PATTERN FOR LOC NETWORK

ARC NAME	FROM NODE	TO NODE	LOWER FLOW BOUND	UPPER FLOW BOUND	FLOW COST	MAX. REPAIR COST	MAX. REPAIR TIME	ARC FLOW	ARC VULNER. PARAM.
ZZAA	ZZ	AA	0	100000	-1000	0	0	5000	0.0
AAA	AA	A	0	100000	0	0	0	1300	0.0
AAB	AA	B	0	100000	0	0	0	1200	0.0
AAC	AA	C	0	100000	0	0	0	1500	0.0
AAD	AA	D	0	100000	0	0	0	1000	0.0
H ZZ	H	ZZ	0	100000	0	0	0	2400	0.0
I ZZ	I	ZZ	0	100000	0	0	0	1800	0.0
J ZZ	J	ZZ	0	100000	0	0	0	400	0.0
L ZZ	L	ZZ	0	100000	0	0	0	400	0.0
B E	B	E	0	1200	5	200	5	1200	0.070
B E	E	B	0	1200	5	200	5	0	0.070
C E	C	E	0	700	6	700	17	0	0.020
C E	E	C	0	700	6	700	17	0	0.020
C F	C	F	0	900	3	600	15	900	0.060
C F	F	C	0	900	3	600	15	0	0.060
C G	C	G	0	600	2	500	13	600	0.030
C G	G	C	0	600	2	500	13	0	0.030
D G	D	G	0	1000	4	300	8	1000	0.050
D G	G	D	0	1000	4	300	8	0	0.050
E H	E	H	0	800	6	400	10	800	0.060
E H	H	E	0	800	6	400	10	0	0.060
E F	E	F	0	500	9	800	20	500	0.020
E F	F	E	0	500	9	800	20	0	0.020
F G	F	G	0	900	3	500	13	0	0.010
F G	G	F	0	900	3	500	13	500	0.010
F H	F	H	0	800	5	750	20	800	0.030
F H	H	F	0	800	5	750	20	0	0.030
F I	F	I	0	1100	7	300	8	1100	0.040
F I	I	F	0	1100	7	300	8	0	0.040
G I	G	I	0	700	5	200	5	700	0.060
G I	I	G	0	700	5	200	5	0	0.060
G J	G	J	0	400	6	400	11	400	0.050
G J	J	G	0	400	6	400	11	0	0.050
A K	A	K	0	900	6	450	18	900	0.090
A K	K	A	0	900	6	450	18	0	0.090
A E	A	E	0	400	3	550	14	400	0.050
A E	E	A	0	400	3	550	14	0	0.050
K E	K	E	0	300	2	350	20	0	0.030
K E	E	K	0	300	2	350	20	300	0.030
K L	K	L	0	400	7	450	25	400	0.030
K L	L	K	0	400	7	450	25	0	0.030
K H	K	H	0	1000	12	550	18	800	0.010
K H	H	K	0	1000	12	550	18	0	0.010

FLOW COST : 61900
THROUGHPUT : 5000

STRIKES TARGETED FOR DAY 1.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
1	A	K	900	831	900	831	2	35	4931	60658	1
2	A	K	831	767	831	767	4	67	4867	59506	2
3	A	K	767	708	767	708	5	96	4808	58444	3
4	A	K	708	654	708	654	5	123	4754	57472	4
5	A	K	654	604	654	604	7	148	4704	56572	5
6	C	E	900	848	900	848	2	183	4652	56052	1
7	A	K	604	558	604	558	8	206	4606	55224	6
8	C	E	848	799	848	799	3	239	4557	54734	2
9	A	K	558	515	558	515	9	260	4514	53960	7
10	C	E	799	752	799	752	3	291	4467	53490	3
11	C	E	752	708	752	708	4	320	4423	53050	4
12	A	K	515	475	515	475	10	340	4383	52330	8
13	C	E	708	667	708	667	5	367	4342	51920	5
14	E	H	800	753	800	753	2	390	4295	51403	1
15	A	K	475	438	475	438	10	408	4258	50737	9
16	C	E	667	628	667	628	6	434	4219	50347	6
17	E	H	753	709	753	709	2	456	4175	49863	2
18	A	K	438	404	438	404	11	473	4141	49251	10
19	C	E	628	591	628	591	6	497	4104	48881	7
20	E	H	709	668	709	668	3	518	4063	48430	3
21	C	E	591	557	591	557	7	541	4029	48090	8
22	A	K	404	373	404	373	12	557	3998	47532	11
23	D	G	1000	951	1000	951	1	572	3949	46846	1
24	E	H	668	629	668	629	3	591	3910	46417	4
25	C	E	557	525	557	525	7	613	3878	46097	9
26	D	G	951	905	951	905	2	627	3832	45453	2
27	E	H	629	592	629	592	4	645	3795	45046	5
28	A	K	373	344	373	344	12	659	3766	44524	12
29	C	E	525	494	525	494	9	679	3735	44214	10
30	D	G	905	861	905	861	2	692	3691	43509	3
31	E	H	592	558	592	558	6	709	3657	43224	6
32	D	G	861	819	861	819	2	722	3615	42636	4
33	C	E	494	465	494	465	9	741	3596	42346	11
34	A	K	344	318	344	318	13	754	3560	41878	13
35	E	H	558	526	558	526	4	770	3528	41526	7
36	D	G	819	779	819	779	3	782	3498	40966	5
37	C	E	465	438	465	438	9	800	3461	40596	12
38	E	H	526	495	526	495	5	815	3430	40255	9

STRIKES TARGETED FOR DAY 1.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
39	A	K	318	294	318	294	13	827	3406	39923	14
40	D	G	779	741	779	741	3	838	3368	39391	6
41	C	E	438	412	438	412	9	855	3342	39131	13
42	F	H	495	466	495	466	5	860	3313	38812	9
43	D	G	741	705	741	705	3	880	3277	38308	7
44	A	K	294	271	294	271	14	891	3254	37896	15
45	C	E	412	388	412	388	10	907	3230	37654	14
46	D	G	705	671	705	671	4	917	3196	37178	8
47	F	H	466	439	466	439	6	931	3169	36881	10
48	A	K	271	250	271	250	14	941	3148	36503	16
49	C	E	388	365	388	365	10	956	3125	36273	15
50	D	G	671	638	671	638	4	966	3092	35811	9

RECAP FOR DAY 1:
****

ARC NAME	FROM	TO	TOTAL NUMBER ARC STRIKES FOR DAY 1	TOTAL NUMBER ARC STRIKES FOR CAMPAIGN
C E	C	E	15	15
D G	D	G	9	9
F H	F	H	10	10
A K	A	K	16	16

NUMBER STRIKES THIS DAY : 50
NUMBER STRIKES THIS CAMPAIGN : 50

ALL ARC REPAIR SCHEDULED FOR DAY 2 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

ARC NAME	FROM NODE	TO NODE	LOWER FLOW BOUND	UPPER FLOW BOUND	FLOW COST	MAX. REPAIR COST	MAX. REPAIR TIME	ARC FLOW	ARC VULNER. PARAM.
ZZAA	ZZ	AA	0	100000	-1000	0	0	3425	0.0
AAA	AA	A	0	100000	0	0	0	701	0.0
AAB	AA	B	0	100000	0	0	0	924	0.0
AAC	AA	C	0	100000	0	0	0	1027	0.0
AAD	AA	D	0	100000	0	0	0	773	0.0
H ZZ	H	ZZ	0	100000	0	0	0	1525	0.0
I ZZ	I	ZZ	0	100000	0	0	0	1100	0.0
J ZZ	J	ZZ	0	100000	0	0	0	400	0.0
L ZZ	L	ZZ	0	100000	0	0	0	400	0.0
B E	B	E	0	1200	5	200	5	924	0.070
C E	C	E	0	427	3	600	15	427	0.060
C G	C	G	0	600	2	500	13	600	0.030
D G	D	G	0	773	4	300	8	773	0.050
E H	E	H	0	524	6	400	10	524	0.060
E F	E	F	0	500	9	800	20	500	0.020
F G	G	F	0	900	3	500	13	273	0.010
F H	F	H	0	800	5	750	20	800	0.030
F I	F	I	0	1100	7	300	8	400	0.040
G I	G	I	0	700	5	200	5	700	0.060
G J	G	J	0	400	6	400	11	400	0.050
A K	A	K	0	301	6	450	18	301	0.090
A E	A	E	0	400	3	550	14	400	0.050
K E	E	K	0	300	2	350	20	300	0.030
K L	K	L	0	400	7	450	25	400	0.030
K H	K	H	0	1000	12	550	18	201	0.010

FLOW COST : 40174
THROUGHPUT : 3425

STRIKES TARGETED FOR DAY 2.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
1	E	H	524	493	524	493	6	981	3394	39833	11
2	D	G	773	735	773	735	4	992	3356	39301	10
3	A	K	301	278	301	278	14	1004	3333	38887	17
4	E	H	493	464	493	464	6	1018	3304	38558	12
5	C	E	427	402	427	402	10	1035	3279	38318	16
6	D	G	735	699	735	699	4	1046	3243	37814	11
7	D	G	699	665	699	665	5	1056	3209	37338	12
8	E	H	464	437	464	437	7	1070	3182	37041	13
9	C	E	402	379	402	379	11	1086	3159	36811	17
10	A	K	278	257	278	257	15	1097	3138	36433	18
11	D	G	665	633	665	633	5	1107	3106	35985	13
12	C	E	379	357	379	357	11	1122	3084	35765	18
13	E	H	437	412	437	412	7	1135	3059	35490	14
14	A	K	257	237	257	237	15	1145	3039	35130	19
15	D	G	633	602	633	602	5	1154	3008	34636	14
16	C	E	357	336	357	336	11	1168	2987	34496	19
17	E	H	412	388	412	388	7	1180	2963	34222	15
18	C	G	600	582	600	582	2	1195	2945	34006	1
19	D	G	602	573	602	573	6	1204	2916	33600	15
20	C	E	336	316	336	316	12	1217	2896	33400	20
21	A	K	237	219	237	219	16	1226	2873	33076	20
22	E	H	388	365	388	365	7	1237	2855	32923	16
23	C	G	582	565	582	565	3	1251	2838	32619	2
24	D	G	573	545	573	545	6	1259	2810	32227	16
25	C	G	565	548	565	548	3	1273	2793	32051	3
26	D	G	545	518	545	518	6	1281	2766	31781	17
27	A	K	219	202	219	202	15	1289	2740	31475	21
28	C	G	548	532	548	532	3	1302	2733	31347	4
29	E	H	365	344	365	344	8	1313	2712	31116	17
30	C	E	316	298	316	298	12	1325	2694	30940	21
31	C	G	532	516	532	516	4	1338	2678	30812	5
32	E	K	300	291	300	291	3	1348	2669	30641	1
33	E	K	291	282	291	282	3	1358	2660	30470	2
34	D	G	518	493	518	493	6	1366	2635	30220	19
35	E	H	344	324	344	324	9	1376	2615	30000	18
36	C	G	516	501	516	501	4	1389	2600	29890	6
37	C	E	298	281	298	281	12	1401	2583	29744	22
38	A	K	202	186	202	186	16	1409	2567	29456	22

STRIKES TARGETED FOR DAY 2.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
39	C	G	501	486	501	486	4	1421	2552	29336	7
40	D	G	493	469	493	469	6	1428	2529	29026	19
41	E	H	324	305	324	305	9	1437	2509	28987	19
42	E	K	292	274	292	274	4	1447	2501	28735	3
43	C	E	281	265	281	265	13	1458	2485	28607	23
44	C	G	486	472	486	472	5	1470	2471	28495	9
45	E	K	274	266	274	266	4	1479	2463	28343	4
46	C	G	472	458	472	458	5	1491	2449	28231	9
47	D	G	459	446	459	446	7	1498	2426	28001	20
48	E	K	266	258	266	258	5	1507	2418	27849	5
49	C	G	458	444	458	444	5	1518	2404	27737	10
50	E	K	258	250	258	250	5	1527	2396	27585	6

RECAP FOR DAY 2:
****

ARC NAME	FROM	TO	TOTAL NUMBER ARC STRIKES FOR DAY 2	TOTAL NUMBER ARC STRIKES FOR CAMPAIGN
C E	C	E	8	23
C G	C	G	10	10
D G	D	G	11	20
E H	E	H	9	19
A K	A	K	6	22
K E	E	K	6	6

NUMBER STRIKES THIS DAY : 50
NUMBER STRIKES THIS CAMPAIGN : 100

ALL ARC REPAIR SCHEDULED FOR DAY 3 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

ARC NAME	FROM NODE	TO NODE	LOWER FLOW BOUND	UPPER FLOW BOUND	FLOW COST	MAX. REPAIR COST	MAX. REPAIR TIME	ARC FLOW	ARC VULNER. PARAM.
ZZAA	ZZ	AA	0	100000	-1000	0	0	2783	0.0
AAA	AA	A	0	100000	0	0	0	636	0.0
AAB	AA	B	0	100000	0	0	0	754	0.0
AAC	AA	C	0	100000	0	0	0	820	0.0
AAD	AA	D	0	100000	0	0	0	573	0.0
H ZZ	H	ZZ	0	100000	0	0	0	1290	0.0
I ZZ	I	ZZ	0	100000	0	0	0	726	0.0
J ZZ	J	ZZ	0	100000	0	0	0	367	0.0
L ZZ	L	ZZ	0	100000	0	0	0	400	0.0
B E	B	E	0	1200	5	200	5	754	0.070
C F	C	F	0	326	3	600	15	326	0.060
C G	C	G	0	494	2	500	13	494	0.030
D G	D	G	0	573	4	300	8	573	0.050
E H	E	H	0	387	6	400	10	387	0.060
E F	E	F	0	500	9	800	20	500	0.020
F H	F	H	0	800	5	750	20	800	0.030
F I	F	I	0	1100	7	300	8	26	0.040
G I	G	I	0	700	5	200	5	700	0.050
G J	G	J	0	400	6	400	11	367	0.050
A K	A	K	0	236	6	450	19	236	0.080
A E	A	E	0	400	3	550	14	400	0.050
K F	E	K	0	267	2	350	20	267	0.030
K L	K	L	0	400	7	450	25	400	0.030
K H	K	H	0	1000	12	550	18	103	0.010

FLOW COST : 31920
THROUGHPUT : 2783

STRIKES TARGETED FOR DAY 3.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
1	A	K	236	219	236	219	17	1536	2765	31596	23
2	E	H	387	364	387	364	9	1547	2742	31343	20
3	D	G	573	545	573	545	7	1555	2714	31053	21
4	C	E	326	307	326	307	13	1568	2695	30873	24
5	D	G	545	518	545	518	7	1576	2668	30603	22
6	A	K	218	201	218	201	17	1584	2651	30297	24
7	E	H	364	343	364	343	9	1595	2630	30066	21
8	C	E	307	289	307	289	13	1607	2612	29908	25
9	D	G	518	493	518	493	7	1615	2587	29658	23
10	E	H	343	323	343	323	9	1625	2567	29438	22
11	C	E	289	272	289	272	14	1636	2550	29302	26
12	C	G	494	470	494	470	6	1648	2535	29192	11
13	D	G	493	469	493	469	7	1655	2511	28942	24
14	A	K	201	186	201	186	17	1663	2495	28672	25
15	E	H	323	304	323	304	9	1672	2477	28463	23
16	C	G	470	465	470	465	6	1684	2453	28351	12
17	C	E	272	256	272	256	14	1695	2447	28223	27
18	D	G	469	446	469	446		1702	2424	27993	25
19	C	G	465	451	465	451	6	1713	2410	27891	13
20	E	K	267	259	267	259	5	1722	2402	27720	7
21	E	K	259	251	259	251	6	1731	2394	27577	8
22	E	H	304	286	304	286	9	1740	2376	27370	24
23	A	K	186	172	186	172	18	1747	2362	27127	26
24	F	E	500	490	500	490	3	1763	2352	26937	1
25	F	E	490	480	490	480	4	1779	2342	26747	2
26	D	G	446	424	446	424	8	1786	2320	26527	26
27	F	E	480	470	480	470	4	1801	2310	26337	3
28	C	G	451	438	451	438	7	1812	2297	26233	14
29	C	E	256	241	256	241	14	1822	2282	26113	28
30	C	G	438	425	438	425	7	1833	2269	26009	15
31	C	G	425	412	425	412	7	1843	2256	25905	16
32	E	H	286	269	286	269	10	1851	2239	25718	25
33	D	G	424	403	424	403	8	1857	2218	25509	27
34	F	K	251	244	251	244	7	1866	2211	25375	9
35	A	K	172	159	172	159	18	1873	2198	25141	27
36	E	K	244	237	244	237	7	1881	2191	25028	10
37	F	E	470	461	470	461	5	1886	2182	24957	4
38	C	E	241	227	241	227	14	1905	2168	24745	29

STRIKES TARGETED FOR DAY 3.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
39	E	F	461	452	461	452	5	1920	2159	24574	5
40	E	K	237	230	237	230	8	1928	2152	24476	11
41	E	F	452	443	452	443	5	1942	2143	24305	6
42	C	G	412	400	412	400	7	1952	2131	24209	17
43	E	K	230	223	230	223	8	1960	2124	24111	12
44	D	G	403	393	403	393	8	1966	2104	23911	28
45	C	F	443	434	443	434	6	1980	2095	23740	7
46	E	H	269	253	269	253	10	1998	2079	23564	26
47	C	G	400	388	400	388	8	1998	2067	23468	18
48	E	F	434	425	434	425	6	2012	2058	23297	9
49	E	K	223	216	223	216	9	2020	2051	23199	13
50	A	K	159	147	159	147	18	2026	2039	23043	28

RECAP FOR DAY 3:
****

ARC NAME	FROM	TO	TOTAL NUMBER ARC STRIKES FOR DAY 3	TOTAL NUMBER ARC STRIKES FOR CAMPAIGN
C F	C	F	5	22
C G	C	G	8	18
D G	D	G	9	28
E H	E	H	7	26
E F	E	F	8	9
A K	A	K	6	28
E K	E	K	7	13

NUMBER STRIKES THIS DAY : 50
NUMBER STRIKES THIS CAMPAIGN : 150

ALL ARC REPAIR SCHEDULED FOR DAY 4 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

ARC NAME	FROM NODE	TO NODE	LOWER FLOW BOUND	UPPER FLOW BOUND	FLOW COST	MAX. REPAIR COST	MAX. REPAIR TIME	ARC FLOW	ARC VULNER. PARAM.
ZZAA	ZZ	AA	0	100000	-1000	0	0	2446	0.0
AAA	AA	A	0	100000	0	0	0	597	0.0
AAB	AA	B	0	100000	0	0	0	618	0.0
AAC	AA	C	0	100000	0	0	0	722	0.0
AAD	AA	D	0	100000	0	0	0	509	0.0
H ZZ	H	ZZ	0	100000	0	0	0	1102	0.0
I ZZ	I	ZZ	0	100000	0	0	0	700	0.0
J ZZ	J	ZZ	0	100000	0	0	0	244	0.0
L ZZ	L	ZZ	0	100000	0	0	0	400	0.0
B F	B	F	0	1200	5	200	5	618	0.070
C F	C	F	0	287	3	600	15	287	0.060
C G	C	G	0	435	2	500	13	435	0.030
D G	D	G	0	509	4	300	8	509	0.050
E H	E	H	0	334	6	400	10	334	0.060
E F	E	F	0	453	9	800	20	453	0.020
F H	F	H	0	800	5	750	20	740	0.030
G I	G	I	0	700	5	200	5	700	0.060
G J	G	J	0	400	6	400	11	244	0.050
A K	A	K	0	197	6	450	18	197	0.080
A E	A	E	0	400	3	550	14	400	0.050
K E	E	K	0	231	2	350	20	231	0.030
K L	K	L	0	400	7	450	25	400	0.030
K H	K	H	0	1000	12	550	18	28	0.010

FLOW COST : 27582
THROUGHPUT : 2446

STRIKES TARGETED FOR DAY 4.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
1	D	G	509	484	509	484	8	2033	2421	27332	29
2	C	F	287	270	287	270	15	2044	2404	27196	30
3	E	H	334	315	334	315	10	2054	2385	26987	27
4	D	G	484	460	484	460	9	2061	2361	26747	30
5	A	K	197	182	197	182	13	2069	2346	26477	29
6	C	F	270	254	270	254	15	2079	2330	26349	31
7	E	H	315	297	315	297	10	2088	2312	26151	28
8	D	G	460	438	460	438	9	2095	2290	25931	31
9	A	K	182	168	182	168	19	2102	2276	25634	30
10	C	F	254	239	254	239	15	2112	2261	25544	32
11	E	H	297	280	297	280	11	2121	2244	25377	29
12	C	G	435	422	435	422	8	2132	2231	25273	19
13	D	G	438	417	438	417	9	2138	2210	25053	32
14	A	K	168	155	168	155	10	2144	2197	24894	31
15	E	H	280	264	280	264	11	2152	2181	24718	30
16	C	F	239	225	239	225	15	2161	2167	24605	33
17	D	G	417	397	417	397	9	2167	2147	24406	33
18	C	G	422	410	422	410	8	2177	2135	24310	20
19	E	F	453	444	453	444	6	2191	2126	24130	9
20	F	K	231	224	231	224	9	2199	2119	24041	14
21	C	G	410	398	410	398	8	2209	2107	23945	21
22	E	F	444	435	444	435	7	2223	2098	23776	10
23	E	F	435	426	435	426	7	2237	2089	23603	11
24	F	K	224	217	224	217	10	2245	2082	23505	15
25	C	G	398	386	398	386	9	2255	2070	23409	22
26	D	G	397	378	397	378	9	2261	2051	23219	34
27	E	H	264	249	264	249	11	2269	2035	23054	31
28	A	K	155	143	155	143	19	2275	2024	22898	32
29	C	F	225	212	225	212	15	2284	2011	22794	34
30	C	G	386	375	386	375	9	2294	2000	22706	23

RECAP FOR DAY 4:
****

ARC NAME	FROM	TO	TOTAL NUMBER ARC STRIKES FOR DAY 4	TOTAL NUMBER ARC STRIKES FOR CAMPAIGN
C F	C	F	5	34
C G	C	G	5	23
D G	D	G	6	24
E H	E	H	5	31
E F	E	F	3	11
A K	A	K	4	32
K F	F	K	2	15

NUMBER STRIKES THIS DAY : 30
NUMBER STRIKES THIS CAMPAIGN : 180

ALL ARC REPAIR SCHEDULED FOR DAY 5 HAS BEEN COMPLETED.
RESULTING FLOW PATTERN IS AS FOLLOWS:

ARC NAME	FROM NODE	TO NODE	LOWER FLOW BOUND	UPPER FLOW BOUND	FLOW COST	MAX. REPAIR COST	MAX. REPAIR TIME	ARC FLOW	ARC VULNER. PARAM.
ZZAA	ZZ	AA	0	100000	-1000	0	0	2406	0.0
AAA	AA	A	0	100000	0	0	0	593	0.0
AAB	AA	B	0	100000	0	0	0	616	0.0
AAC	AA	C	0	100000	0	0	0	694	0.0
AAD	AA	D	0	100000	0	0	0	503	0.0
H ZZ	H	ZZ	0	100000	0	0	0	1081	0.0
I ZZ	I	ZZ	0	100000	0	0	0	700	0.0
J ZZ	J	ZZ	0	100000	0	0	0	225	0.0
L ZZ	L	ZZ	0	100000	0	0	0	400	0.0
B E	B	E	0	1200	5	200	5	616	0.070
C E	C	E	0	272	3	600	15	272	0.060
C G	C	G	0	422	2	500	13	422	0.030
D G	D	G	0	503	4	300	8	503	0.050
E H	E	H	0	330	6	400	10	330	0.060
E F	E	F	0	453	9	800	20	453	0.020
F H	F	H	0	800	5	750	20	725	0.030
G I	G	I	0	700	5	200	5	700	0.060
G J	G	J	0	400	6	400	11	225	0.050
A K	A	K	0	193	6	450	18	193	0.080
A F	A	F	0	400	3	550	14	400	0.050
K F	F	K	0	233	2	350	20	233	0.030
K L	K	L	0	400	7	450	25	400	0.030
K H	K	H	0	1000	12	550	18	26	0.010

FLOW COST : 27220
THROUGHPUT : 2406

STRIKES TARGETED FOR DAY 5.

MISSION NUMBER	FROM	TO	ARC CAPACITY BEFORE STRIKE	ARC CAPACITY AFTER STRIKE	ARC FLOW BEFORE STRIKE	ARC FLOW AFTER STRIKE	TO BE REPAIRED ON DAY	CUM. REPAIR COST	MAXIMUM NETWORK FLOW	NETWORK FLOW COST	NUMBER OF ARC STRIKES
1	D	G	503	478	503	478	9	2301	2381	26970	35
2	E	H	330	311	330	311	11	2311	2362	26761	32
3	A	K	193	178	193	178	19	2319	2347	26491	33
4	D	G	478	455	478	455	9	2325	2324	26261	36
5	C	E	272	256	272	256	16	2336	2308	26123	35
6	E	H	311	293	311	293	11	2345	2290	25935	33
7	D	G	455	433	455	433	10	2352	2269	25715	37
8	A	K	178	164	178	164	20	2359	2254	25478	34
9	C	E	256	241	256	241	16	2369	2239	25358	36
10	E	H	293	276	293	276	12	2379	2222	25171	34
11	D	G	433	412	433	412	10	2384	2201	24961	38
12	C	E	241	227	241	227	16	2393	2187	24849	37
13	A	K	164	151	164	151	20	2399	2174	24680	35
14	C	G	422	410	422	410	9	2409	2162	24584	24
15	D	G	412	392	412	392	10	2415	2142	24384	39
16	E	H	276	260	276	260	12	2423	2126	24208	25
17	E	F	453	444	453	444	7	2437	2117	24037	12
18	F	K	233	226	233	226	10	2445	2110	23939	16
19	C	G	410	398	410	398	9	2455	2098	23843	25
20	E	F	444	435	444	435	8	2469	2089	23672	13
21	F	K	226	219	226	219	10	2477	2082	23574	17
22	E	F	435	426	435	426	8	2491	2073	23403	14
23	C	G	398	386	398	386	10	2501	2061	23307	26
24	D	G	392	373	392	373	10	2507	2042	23117	40
25	C	E	227	214	227	214	16	2516	2029	23013	38
26	E	H	260	245	260	245	12	2524	2014	22848	36
27	A	K	151	139	151	139	20	2530	2002	22692	36
28	C	G	386	375	386	375	10	2540	1991	22604	37

RECAP FOR DAY 5:
****

ARC NAME	FROM	TO	TOTAL NUMBER ARC STRIKES FOR DAY 5	TOTAL NUMBER ARC STRIKES FOR CAMPAIGN
C E	C	E	4	38
C G	C	G	4	27
D G	D	G	6	40
E H	E	H	5	36
E F	E	F	3	14
A K	A	K	4	36
K F	F	K	2	17

NUMBER STRIKES THIS DAY : 28
NUMBER STRIKES THIS CAMPAIGN : 209

BIBLIOGRAPHY

1. Danskin, J. M., "Combat Problems, Weapons Systems, and the Theory of Allocation," Journal of the Operations Research Society of America, v. 2, p. 204-205, March 1954.

2. Danskin, J. M., Theory of Max-min and its Applications to Weapons Allocations Problems, Springer-Verlag New York Inc., 1967.

3. Durbin, E. P., An Interdiction Model of Highway Transportation, The Rand Corporation, R-4945-PR, May 1966.

4. Ford, L. R., Jr., and Fulkerson, D. R., Flows in Networks, The Rand Corporation, R-375-PR, August 1962.

5. Holliday, L. P., A Method for Estimating Road Capacity and Truck Requirements (U), The Rand Corporation, RM-3331, ARPA, November 1963 (Confidential).

6. Koopman, B. O., Search and Screening, Operations Evaluation Group, Office of the Chief of Naval Operations, Navy Department, 1946.

7. Koopman, B. O., "The Optimum Distribution of Effort," Journal of the Operations Research Society of America, v. 1, p. 52-63, January 1953.

8. Hancock, H., Theory of Maxima and Minima, Ginn and Company, 1917.

9. Mustin, T. M., Optimal Allocation of Airstrikes for Interdiction of a Transportation Network, M.S. Thesis, Naval Postgraduate School, 1967.

10. Nugent, R. O., The Optimum Allocation of Airstrikes Against a Transportation Network for an Exponential Damage Function, M.S. Thesis, Naval Postgraduate School, 1969.

11. Wollmer, R. D., Removing Arcs from a Network, Operations Research Center, University of California, Berkeley, ORC64-5 (RR), 13 March 1964.

12. Wollmer, R. D., Algorithms for Targeting Strikes in a Lines-of-Communication (LOC) Network, The Rand Corporation, R-5864-PR, February 1969.

13. Wollmer, R. D., and Ondrasek, M. J., A Model for Targeting Strikes in an LOC Network, The Rand Corporation, RM-5940-PR, September 1969.

14. Wollmer, R. D., Sensitivity Analysis of Networks, Operations Research Center, University of California, Berkeley, ORC65-8, April 1965.

15. Wollmer, R. D., Some Methods for Determining the Most Vital Link in a Railway Network, The Rand Corporation, RM-3321-ISA, April 1963.

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	2
2.	Library, Code 0212 Naval Postgraduate School Monterey, California 93940	2
3.	Department of Operations Analysis, Code 55 Naval Postgraduate School Monterey, California 93940	1
4.	Asst. Professor James G. Taylor, Code 55Tw Department of Operations Analysis Naval Postgraduate School Monterey, California 93940	1
5.	Major Larry R. Capps Student Officer Company USA Ordnance Center and School Aberdeen Proving Ground, Maryland 21005	1

Security Classification

DOCUMENT CONTROL DATA - R & D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) Naval Postgraduate School Monterey, California 93940	2a. REPORT SECURITY CLASSIFICATION Unclassified 2b. GROUP

3. REPORT TITLE

A Dynamic Model for the Allocation of Airstrikes Against a Lines-of-Communication Network.

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; September 1970

5. AUTHOR(S) (First name, middle initial, last name)

Larry Reginald Capps

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
September 1970	66	15

8a. CONTRACT OR GRANT NO. b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. DISTRIBUTION STATEMENT

This document has been approved for public release and sale; its distribution is unlimited.

11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY
	Naval Postgraduate School Monterey, California 93940

13. ABSTRACT

A computer model is presented for determining the daily allocation of airstrikes for interdicting a lines-of-communication (LOC) network assuming an exponential damage function. The allocation of sorties to arcs in the network is based on the assumptions that (1) flow of supplies is restricted by network capacity and (2) parameter values for each arc are independent. Upper and lower flow bounds, cost per unit flow, maximum repair time and cost, and the vulnerability parameter for each arc are required as input data. The model selects that arc to strike which maximizes the repair cost plus the product of the increase in minimum cost circulation flow and repair time. The procedure is programmed in daily cycles, allowing repair on interdicted arcs. A sample problem and all documentation necessary for duplicating the computer program are given.

DD FORM 1 NOV 65 1473 (PAGE 1)

S/N 0101-807-6811

Security Classification

A-31408

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Lines-of-Communication Network						
Allocation of Airstrikes						
Exponential Damage Function						
Interdiction						
Computer Model						

Thesis 120960
C1953 Capps
c.1

A dynamic model for the allocation of air-strikes against a lines-of-communication network.